A Refinement of Mindlin Plate Theory Using Simultaneous Rotary Inertia and Shear Correction Factors

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We revisit Mindlin's theory for flexural dynamics of plates using two correction factors, one for shear and one for rotary inertia. Mindlin himself derived and considered his equations with both correction factors, but never with the two simultaneously. Here, we derive optimal values of both factors by matching the Mindlin frequency–wavenumber branches with the exact Rayleigh–Lamb dispersion relations. The thickness shear resonance frequency is obtained if the factors are proportional but otherwise arbitrary. This degree-of-freedom allows matching of the main flexural mode dispersion with the exact Lamb wave at either low or high frequency by choosing the shear correction factor as a function of Poisson's ratio. At high frequency, the shear factor takes the value found by Mindlin, while at low frequency, it assumes a new explicit form, which is recommended for flexural wave modeling. [DOI: 10.1115/1.4038956]

1 Introduction

The equations for flexural wave motion in thin plates first proposed by Uflyand [1] were subsequently shown by Mindlin [2] to follow from Hamilton's principle with kinetic and energy densities derived from the full elastodynamic equations of motion. Mindlin later generalized his procedure for reduced order Lagrangian density to obtain higher-order theories [3,4] that more accurately reflect plate dynamics at shorter wavelength. Here, we focus on the simplest and most succinct form of this hierarchy of equations [1,2], which we refer to as Mindlin plate theory. Mindlin's theory incorporates shear deformation and rotary inertia; it predicts finite wave speed for the primary flexural branch as frequency tends to infinity, and it also displays a second quasi-flexural branch that approximates the analogous Rayleigh–Lamb branch, all features absent from the classical Kirchhoff theory. Application of the Mindlin equations has enabled accurate modeling of flexural wave effects not possible with Kirchhoff theory, such as plate edge waves [5] and scattering of flexural waves from defects [6].

In his original paper [2] on flexural wave motion, Mindlin considered two possible values for an adjustable parameter in his theory, the shear correction factor or coefficient. He showed that the flexural wave matches the Rayleigh wave speed at high frequency with a shear correction factor that depends on Poisson's ratio, while the lowest shear resonance frequency is matched using a factor independent of Poisson's ratio. Hutchinson [7] proposed a third shear correction factor, which is a function of Poisson's ratio that matches the flexural wave with the Rayleigh–Lamb mode to second-order in frequency. Stephen [8] investigated the accuracy of Hutchinson's factor in predicting the long-wavelength Rayleigh–Lamb mode and judged it the "best" correction factor. The choice of which value to choose depends upon the application in mind, with no unique shear correction factor optimal for every problem. Thus, Hull [9] noted that in order to properly match the dispersion of the Rayleigh–Lamb solution, the shear correction factor must be a fully frequency-dependent parameter, which is not of much use in practice when one wants a theory with as few adjustable parameters as possible. Recently, Lakawicz and Bottega [10] showed that two distinct shear correction factors can replicate the three lowest antisymmetric Rayleigh–Lamb branches. While this is a novel approach, it suffers from the fact that the displacement solution comprises selected parts of the total solution using Mindlin's equations with different parameters. That is, the two types of solutions used are not the consequence of a unique Lagrangian.

Mindlin's theory [4] actually includes two adjustable parameters in the equations of motion. The first is the shear correction factor which was originally motivated by the need to model shear stress more accurately. The second is the rotary inertia correction factor, which is perhaps not as well known as the shear factor. The inertia factor arises from the angular acceleration terms of the reduced order equations of motion, which allows for the density in the angular acceleration to differ from the actual density of the plate. Mindlin proposed a modified density to account for the error in the prediction of the model[1] [4]: "Thus we can correct the limiting frequencies of the upper modes by replacing ρ_1 in the kinetic energy-density by ρ/κ^2 where $\kappa = \pi/\sqrt{12}$, or by replacing the $S_{2j}^{(0)}$ in the strain-energy by $\kappa S_{2j}^{(0)}$." Mindlin apparently viewed the modified density on an equal footing with the shear coefficient, in that either one provides the necessary correction. Considering the success of his theory using a single correction factor, it is not surprising that he did not investigate the consequences of using two independent correction factors for shear and rotary inertia. However, it is remarkable that there does not seem to be any subsequent examination in the literature of the potential of using two independent correction factors. As far as the author can ascertain, the sole exception is a passing suggestion in favor of such an approach by Benscoter [11] in a review of a paper by Mindlin and Deresiewicz [12] on the shear correction term in the Timoshenko beam theory. Mindlin was certainly not the first to incorporate rotary inertia in flexural vibration; it had been considered as early as 1859 by Bresse [13]. But he was the first to consider a *modified rotary inertia*, which we take advantage of.

The purpose here is to propose a refinement of Mindlin's equations that incorporates *independent* correction factors for shear and for angular acceleration. We start with the fact that the upper mode cut-on frequencies are corrected by either the use of a shear correction factor κ or a density correction factor λ. Thus, Mindlin and Yang [4] noted that either $\kappa = \pi^2/12$ with $\lambda = 1$ or $\kappa = 1$ and $\lambda = 12/\pi^2$ yields the correction. However, it has apparently not been noted that the identity $\kappa/\lambda = \pi^2/12$ suffices, which allows flexibility in choosing one or the other of κ or λ. We use this flexibility to show that both the shear resonance and the low frequency matching can be obtained with a unique choice of κ, which we propose as a new best choice for the shear correction factor.

The paper proceeds with a review of the Mindlin theory in Sec. 2. The various shear correction factors mentioned above are introduced in Sec. 3. The improved accuracy using two correction factors is explored in Sec. 4, where the optimal shear and rotary inertia correction factors are defined and justified by comparison with previous models. Section 5 concludes the paper.

2 Mindlin Equations

The plate has thickness h, density ρ, and the isotropic elastic parameters are shear modulus μ, Poisson's ratio ν, and Young's modulus $E = 2(1+\nu)\mu$. The moment of inertia and bending stiffness are $I = h^3/12$ and $D = EI/(1-\nu^2)$, respectively. The remaining parameter in Mindlin's theory is the shear correction factor κ, which is introduced to better approximate the shear forces. It may be chosen according to different criteria, but normally $\kappa < 1$ [14].

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[1]The parameter κ used in this paper corresponds to Mindlin's κ^2 [2–4].

The two-dimensional (2D) Mindlin equations are formulated in terms of $\mathbf{x} = (x_1, x_2)$ the position on the central plane of the plate. The kinematic variables are [14] the vertical plate deflection $w(\mathbf{x}, t)$ and the in-plane two-vector of rotations $\boldsymbol{\psi}(\mathbf{x}, t)$. Define the tensor $\boldsymbol{\varepsilon}$, or 2×2 matrix of elements ε_{ij}, $i, j = 1, 2$, $\boldsymbol{\varepsilon} = \frac{1}{2}(\nabla\boldsymbol{\psi} + (\nabla\boldsymbol{\psi})^{\mathrm{T}})$. The strain energy density per unit area of the plate is, see e.g., Ref. [15]

$$U = \frac{D}{2}\left(\nu(\operatorname{tr}\boldsymbol{\varepsilon})^2 + (1-\nu)\operatorname{tr}(\boldsymbol{\varepsilon}^2)\right) + \frac{\kappa}{2}\mu h|\nabla w + \boldsymbol{\psi}|^2 \tag{1}$$

The kinetic energy density is taken in the form

$$T = \frac{1}{2}\rho h \dot{w}^2 + \frac{1}{2}\rho_1 I |\dot{\boldsymbol{\psi}}|^2 \tag{2}$$

where the dot over a quantity indicates the time derivative. The modified density ρ_1 is related to but different from ρ, in accordance with Mindlin's first-order approximation [4]. Application of Hamilton's principle leads to the equations of motion [1,2]

$$\operatorname{div}\mathbf{Q} = \rho h \ddot{w} \tag{3}$$

$$\operatorname{div}\mathbf{M} - \mathbf{Q} = \rho_1 I \ddot{\boldsymbol{\psi}} \tag{4}$$

where $\mathbf{M}$ and $\mathbf{Q}$ are, respectively, the bending moment tensor (or 2×2 matrix) and the shear force (two-vector)

$$\mathbf{M} = D(\nu(\operatorname{tr}\boldsymbol{\varepsilon})\mathbf{I} + (1-\nu)\boldsymbol{\varepsilon}) \tag{5}$$

$$\mathbf{Q} = \kappa\mu h(\nabla w + \boldsymbol{\psi}) \tag{6}$$

and $\mathbf{I}$ is the identity ($\mathrm{I}_{ij} = \delta_{ij}$).

Motivation for the modified density ρ_1 comes from the fact that displacement at any point (x, y, z) in the plate according to Mindlin's theory is $\mathbf{u} = z\,\boldsymbol{\psi}(\mathbf{x}, t) + w(\mathbf{x}, t)\,\mathbf{e}_z$, where $\mathbf{x} = (x, y)$ is the two-dimensional position on the central plane of the plate, z is the transverse coordinate through the plate with $z = 0$ the center plane. The actual displacement for the elastodynamic solution differs from this, and just as one might expect the shear to be better modeled with the shear correction factor, by analogy the rotational inertia requires its own distinct correction factor. We introduce it as λ, according to the definition

$$\rho_1 = \lambda\rho \tag{7}$$

We consider time harmonic motion of frequency ω, for which the most general solution of the homogeneous equations of motion is of the form

$$\begin{aligned} w &= \operatorname{Re}\{(v_1 + v_2)e^{-i\omega t}\} \\ \boldsymbol{\psi} &= \operatorname{Re}\{(\beta_1\nabla v_1 + \beta_2\nabla v_2 - \mathbf{e}_z \times \nabla v_3)e^{-i\omega t}\} \end{aligned} \tag{8}$$

where $v_1(\mathbf{x})$, $v_2(\mathbf{x})$, and $v_3(\mathbf{x})$ (see Ref. [2]) each satisfies its own (two-dimensional) Helmholtz equation

$$\nabla^2 v_j + k_j^2 v_j = 0, \quad j = 1, 2, 3 \quad (\text{no summation}) \tag{9}$$

The three bulk wavenumbers k_1, k_2, and k_3 and the numbers β_1 and β_2 depend upon ω. They are followed by direct substitution and are given by

$$\left.\begin{matrix} k_1^2 \\ k_2^2 \end{matrix}\right\} = \frac{1}{2}\left(\frac{k_T^2}{\kappa} + \lambda k_P^2\right) \pm \sqrt{\frac{1}{4}\left(\frac{k_T^2}{\kappa} - \lambda k_P^2\right)^2 + k_F^4} \tag{10a}$$

$$k_3^2 = \lambda k_T^2 - \kappa h / I \tag{10b}$$

where k_T and k_P are, respectively, the wavenumbers for transverse and extensional waves [16], and k_F is the wavenumber according to Kirchhoff plate theory

$$k_T = \omega\sqrt{\frac{\rho}{\mu}}, \quad k_P = \omega\sqrt{\frac{(1-\nu^2)\rho}{E}}, \quad k_F^2 = \omega\sqrt{\frac{\rho h}{D}} \tag{11}$$

Finally, the nondimensional parameters appearing in Eq. (8) are $\beta_j = -1 + k_T^2/(\kappa k_j^2), j = 1, 2$.

3 Single Correction Factors

3.1 Low Frequency Correction Factor. Expansion of the expression (10a) about $\omega = 0$ yields as the leading term the classical Kirchhoff wavenumber $k_1^2 = k_F^2$, which is of order ω. The next term in the expansion of k_1^2 is of order ω^2 with coefficient proportional to $1/\kappa + \lambda(1-\nu)/2$. Previous authors have noted that Mindlin's theory with the shear correction factor only ($\lambda = 1$) can reproduce the low frequency expansion of the A_0 Lamb wave (denoted the F_1 mode here) with a suitable choice, viz. $\kappa = 5/(6-\nu)$ [8]. It is clear from the term of order ω^2 that the same correction is obtained using an inertia correction factor only, and equal to 7/5. In summary, the low frequency condition is satisfied by a either a shear or an inertial correction factor alone, i.e., $(\kappa, \lambda) = (\kappa_1, 1)$ or $(\kappa, \lambda) = (1, \lambda_1)$, respectively, where

$$\kappa_1 = \frac{5}{6-\nu}, \quad \lambda_1 = \frac{7}{5} \tag{12}$$

Figure 1 plots the three Mindlin dispersion curves along with the exact Rayleigh–Lamb branches. The accuracy of the approximation with the F_1 wave is evident from the relative error shown in Fig. 2. The dispersion equation for antisymmetric Rayleigh–Lamb modes is [14]

$$\tan\frac{\gamma_L h}{2}\cot\frac{\gamma_T h}{2} + 4\gamma_L\gamma_T k^2\left(\gamma_T^2 - k^2\right)^{-2} = 0 \tag{13}$$

with $\gamma_T = \sqrt{k_T^2 - k^2}$, $\gamma_L = \sqrt{k_L^2 - k^2}$ where k_L is the longitudinal wavenumber, $k_T^2/k_L^2 = 1 + 1/(1-2\nu)$.

3.2 Shear Wave Correction Factor. The exact shear branch (S) wavenumber k_S is given by

$$k_S^2 = k_T^2 - \pi^2/h^2 \tag{14}$$

The cut-on frequency $k_T = \pi/h$ where $k_S = 0$ is also a zero for the exact elastodynamic mode corresponding to F_2. Noting the identity $k_1^2 k_2^2 = k_3^2 k_P^2/\kappa$, it follows that both k_2 and k_3 are zero at the cut-on frequency $k_T^2 = \kappa h/(\lambda I)$, analogous to the double zero for the two exact modes. As noted by Mindlin and Yang [4], this

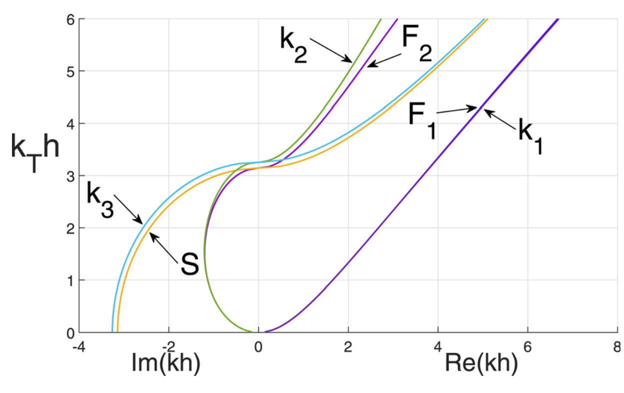

Fig. 1 The wavenumbers of the three exact branches, F_1, F_2, S, and the Mindlin wavenumbers k_1, k_2, k_3 from Eq. (10) for $(\kappa, \lambda) = (\kappa_1, 1)$, with $\nu = 0.35$ The abscissa shows the real and imaginary parts of the nondimensional wavenumber kh for the separate branches. The ordinate is nondimensional frequency defined by k_T of Eq. (11).

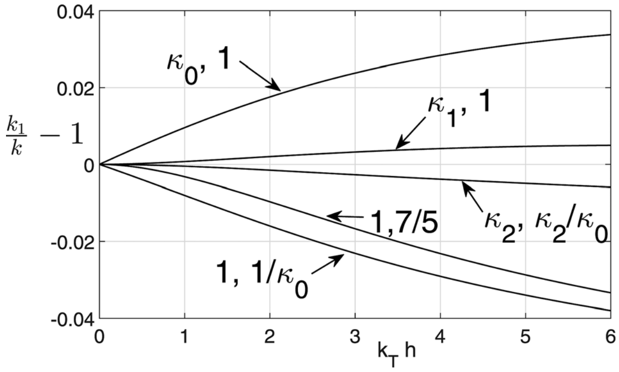

Fig. 2 The relative error in the F_1 flexural wavenumber k_1 compared with the exact Rayleigh–Lamb wavenumber k, as a function of nondimensional frequency k_{Th} for five combinations of κ, λ. The curves represent k_1 calculated from Eq. (10*a*) with shear correction factors only and with both correction factors κ, λ satisfying the constraint (17).

condition is satisfied by a either a shear or an inertial correction factor alone, i.e., $(\kappa, \lambda) = (\kappa_0, 1)$ or $(\kappa, \lambda) = (1, 1/\kappa_0)$, respectively, where

$$\kappa_0 \equiv \pi^2/12 \tag{15}$$

Figure 3 shows that the S-branch dispersion curve is exactly replicated by k_3. However, the agreement of k_1 and k_2 with the F_1 and F_2 branches is not so good at higher frequencies.

3.3 High Frequency Correction Factor. At high frequency, Eq. (10) gives

$$(k_1, k_2, k_3) = \left(\frac{k_T}{\sqrt{\kappa}}, \sqrt{\lambda} k_P, \sqrt{\lambda} k_T\right) + \cdots \tag{16}$$

where only the leading order terms are indicated. Mindlin [2] pointed out that the F_1 branch has the correct asymptote at high frequency, $k_1 \to k_R = \omega/c_R$ where c_R is the Rayleigh wave speed, using the shear correction factor $\kappa = \kappa_R \equiv c_R^2/c_T^2$. Note that κ_R is the positive root less than unity of the cubic polynomial [17] $s^3 - 8s^2 + 8[(2-\nu)s - 1]/(1-\nu)$.

Unlike the case with the low frequency response, it is not possible to obtain the F_1 high frequency asymptote using only an inertial correction factor. However, the latter does provide the possibility of simultaneously correcting both the shear and quasi-shear branches, as we will see below.

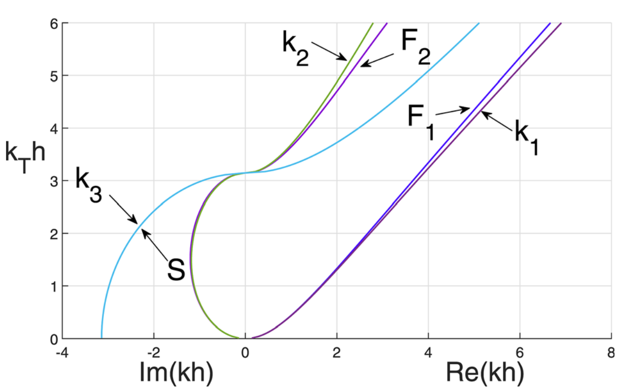

Fig. 3 Exact and Mindlin wavenumbers for $(\kappa, \lambda) = (\kappa_0, 1)$, $\nu = 0.35$ The abscissa and ordinate are the same as in Fig. 1

4 The Two Correction Factors

As a first criterion, we choose the correction factors such that the Mindlin shear cut-on coincides with the exact cut-on frequency. Based on Eqs. (10*b*) and (14), this is achieved with

$$\frac{\kappa}{\lambda} = \frac{\pi^2}{12} \tag{17}$$

which is assumed hereafter. Only one parameter remains to be fixed, say κ, in terms of which

$$\lambda = \frac{\kappa}{\kappa_0} \tag{18}$$

The three wavenumbers are now

$$\left.\begin{matrix} k_1^2 \\ k_2^2 \end{matrix}\right\} = \frac{1}{2}\left(\frac{k_T^2}{\kappa} + \kappa\frac{k_P^2}{\kappa_0}\right) \pm \sqrt{\frac{1}{4}\left(\frac{k_T^2}{\kappa} - \kappa\frac{k_P^2}{\kappa_0}\right)^2 + k_F^4} \tag{19a}$$

$$k_3^2 = \frac{\kappa}{\kappa_0} k_S^2 \tag{19b}$$

Note that the wavenumbers k_2 and k_3 are pure imaginary below the shear cut-on at $k_T = \pi/h$ and real above that frequency.

4.1 Low Frequency Correction Factor. The present model emulates the low frequency behavior of the F_1 branch if the following holds:

$$\frac{1}{\kappa} + \lambda\frac{(1-\nu)}{2} = \frac{6-\nu}{5} + \frac{1-\nu}{2} \tag{20}$$

Eliminating λ using Eq. (18) yields a quadratic equation for the shear correction factor with two real roots: one always less than unity, the other greater than two. Of the two roots for κ, we find that the larger is not realistic for reasons explained further below, and therefore focus on the smaller one, $\kappa = \kappa_2$, where

$$\kappa_2 \equiv \frac{20}{17 - 7\nu}\left(1 + \sqrt{1 - \frac{200(1-\nu)}{\kappa_0(17-7\nu)^2}}\right)^{-1} \tag{21}$$

The shear correction factor κ_2 is shown in Fig. 4 for positive Poisson's ratio. Over this range of ν, the factor κ_2 is always greater than the standard low frequency shear correction factor κ_1, and the associated values of λ are always less than unity since $\kappa \geq \kappa_0$. Note that κ_0, κ_1, and κ_2 all coincide at $\nu = 6 - 60/\pi^2 = -0.079$. Figure 4 also shows the high frequency correction factor κ_R. The associated density correction factor $\lambda_R = \kappa_R/\kappa_0$ is larger than unity for Poisson's ratio values less than approximately 0.175, but otherwise $\lambda_R < 1$.

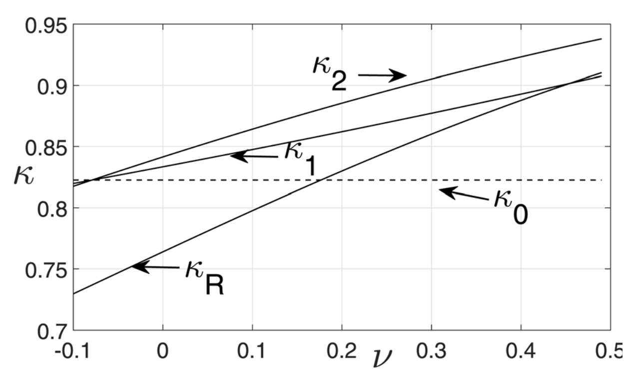

Fig. 4 The shear correction factors κ_0, κ_1, κ_2 from Eqs. (12), (15), and (21), respectively, and the high frequency F_1 correction factor κ_R, as functions of Poisson's ratio ν

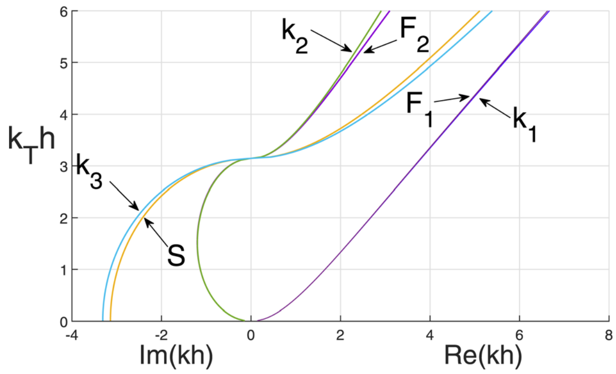

Fig. 5 The exact and Mindlin wavenumbers for $(\kappa, \lambda) = (\kappa_2, \kappa_2/\kappa_0)$, with $\nu = 0.35$ The abscissa and ordinate are the same as in Fig. 1

Figure 5 shows the Mindlin dispersion curves calculated using shear correction factor κ_2 with inertial correction factor satisfying the shear resonance criterion (18). The agreement between the k_1 and F_1 curves is much more accurate than that of Fig. 3 and is in fact as good as that obtained using the low frequency shear correction factor κ_1, as the close-up in Fig. 2 indicates. However, it is important to emphasize that the dispersion curves of Fig. 5 also have the correct shear resonance (cut-on) at the exact value, i.e., $k_T h = \pi$. Furthermore, the agreement between the k_2 and F_2 curves is decidedly better in Fig. 5 than in either of Figs. 1 or 3. Finally, we note that the alternative root of the quadratic equation for κ, from Eqs. (18) to (20), viz.

$$\kappa = \frac{20}{17 - 7\nu}\left(1 - \sqrt{1 - \frac{200(1-\nu)}{\kappa_0(17-7\nu)^2}}\right)^{-1} \tag{22}$$

produces exactly the same k_1 and k_2 curves as shown in Fig. 5. However, the predicted k_3 branch is highly inaccurate. As mentioned above, the alternative value (22) is always larger than two, which is clearly unrealistic, apart from the wave dispersion implications.

5 Conclusions

Mindlin himself considered both shear and rotary inertial correction factors, but never the two simultaneously. The main contribution of this paper is to show that a consistent low frequency theory that emulates the flexural wave and reproduces the exact shear thickness resonance can be obtained with unique values of the two correction factors. Our first result is that the thickness shear resonance frequency is obtained if the shear and rotary inertial correction factors in Mindlin's equations are proportional: $\kappa = \lambda\pi^2/12$. The remaining degree-of-freedom allows matching of the flexural mode dispersion with the exact Lamb wave at either low or high frequency by choosing κ as a function of Poisson's ratio. At high frequency, the shear correction factor takes the value found by Mindlin [2], while at low frequency, it has a new explicit form, κ_2 of Eq. (21). The results of the paper suggest that Mindlin's equations are optimal in the long wavelength regime with $\kappa = \kappa_2$ and $\lambda = 12\kappa_2/\pi^2$. It is recommended to use these specific values for the two correction factors when using Mindlin's theory for modeling plate dynamics.

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