Comments on "An analytical study of sound transmission through unbounded panels of functionally graded materials doi:10.1016/j.jsv.2010.09.020"

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The vector function in Eq. (28) which is presented as the solution of a system of ordinary differential equations is incorrect.

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I. INTRODUCTION

The problem considered in[1] is reduced to finding the solution of a system of ordinary differential equations, Eq. (26):

$$\frac{\mathrm{d}\,\mathbf{u}}{\mathrm{d}\,z} = \mathbf{A}(z)\mathbf{u}. \tag{1}$$

The 6-vector $\mathbf{u}$ comprises the displacement and the traction 3-vectors, and $\mathbf{A}(z)$ is a 6×6 matrix. The paper claims that the solution for Eq. (26) can be written as

$$\mathbf{u}(z) = \mathrm{e}^{\int_0^z \mathbf{A}(s)\,\mathrm{d}\,s}\,\mathbf{u}(0). \tag{2}$$

The transfer matrix $\mathbf{T}$ is defined in Eq. (29) such that $\mathbf{u}(h) = \mathbf{T}\mathbf{u}(0)$. Consider $h = 2$, $\mathbf{A}(s) = \mathbf{Y}, \mathbf{X}$ for $s \in [0,1), [1,2]$, respectively. The transfer function according to[1] is $\mathbf{T} = \mathrm{e}^{\mathbf{X}+\mathbf{Y}}$, but the correct solution is $\mathbf{T} = \mathrm{e}^{\mathbf{X}}\,\mathrm{e}^{\mathbf{Y}}$. The matrices $\mathrm{e}^{\mathbf{X}}\,\mathrm{e}^{\mathbf{Y}}$, $\mathrm{e}^{\mathbf{Y}}\,\mathrm{e}^{\mathbf{X}}$ and $\mathrm{e}^{\mathbf{X}+\mathbf{Y}}$ are distinct for non-commutative $\mathbf{X}$ and $\mathbf{Y}$. The relation between these exponentials is at the heart of the Baker-Campbell-Hausdorff formula.

In the general case of an arbitrary piecewise continuous $\mathbf{A}(s)$ with appropriate auxiliary conditions at the points of discontinuity, the transfer matrix $\mathbf{T}$ is given by the multiplicative integral of $\mathbf{A}(s)$ which is evaluated by the Peano series. This solution to Eq. (1) is exact and unique. It is well documented in mathematical textbooks, e.g.[2]. Its application to acoustics of layered and functionally graded materials is discussed in detail in[3,4] for rectangular and in[5] for cylindrical coordinates.

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