Acoustic and flexural wave scattering from a three-member junction

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A three-member junction is formed by a pair of semi-infinite plates in contact with fluid on one side and a mechanical structure on the other. The latter is described by an impedance matrix. The excitation is either a straight crested flexural wave traveling on one of the fluid-loaded plates and obliquely incident on the line junction, or an acoustic plane-wave incident from any direction in the fluid. The general solution for this type of scattering problem is derived and illustrative numerical examples are given. The admittance matrix for the fluid-loaded plate junction without the attachment plays a central role in the solution. It is verified that the general solution reduces to that for a pair of plates with clamped and welded junction conditions as limiting cases when the frame impedance is zero and infinite. The numerical results display two well-defined characteristic critical angles for transmission of structural energy and diffraction of acoustic pressure. © *1995 Acoustical Society of America.*

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INTRODUCTION

Discontinuities and reinforcements play a crucial role in enhancing the amount of energy scattered and radiated from fluid-loaded structures. The analysis of a single scatterer or obstruction, acoustically isolated from others on the structure, contains the essential features of the problem and provides some quantitative information about the processes involved. Thus some time ago, Lyon[1] obtained simple approximations for the sound radiated from a beam attached to a plate. This work was subsequently complemented by the calculation of the force and moment admittances, obtained numerically by Nayak[2] and analytically by Crighton,[3] with further insights given by Smith.[4] This was followed by comprehensive studies of transmission and acoustic scattering from a single rib on a panel.[5–7] Recently, the effects of varying junction conditions between the rib and plate have been studied by Guo.[8,9] We note that all these analyses concerned a reinforcement attached to a uniform panel.

In this paper we consider the more general but practically realistic situation of a nonuniform structure with a reinforcing member. The structural configuration is modeled as a single rib attached to the junction of two dissimilar plates. The three-member junction is depicted in Fig. 1. We will investigate the consequences of the three-member junction on an obliquely incident wave, either acoustic or structural, although the numerical examples will focus on structural wave incidence. The related problem of a wave obliquely incident on a rib on a uniform plate was analyzed by Lyapunov[10] and more recently by Photiadis.[11] Oblique incidence was also briefly discussed by Crighton and Maidanik[6] for the case of a rib attached to a membrane. In general, the acoustic effect of an attached internal rib on a uniform plate can be completely described by the line admittance (or impedance) matrix for the fluid-loaded plate. This is the matrix relating force and moment to the plate deflection and rotation at the drive location, and is diagonal for a uniform plate,[11,12] as can be easily seen from symmetry arguments. The reinforcement acts as a reaction load on the uniform plate, which can be directly determined using the plate admittance matrix and the impedance matrix for the rib. In this way one solves the scattering problem by a standard superposition of forces.

The three-member problem of Fig. 1, on the other hand, cannot be solved in exactly the same manner. Consider a flexural wave incident from plate 1 striking the dissimilar plate junction with an equivalent applied force and moment replacing the rib. No matter how one chooses the effective forces and moments at the junction, the equivalent line load cannot cancel the incident wave on the second plate, plate 2, as it must. This is because plate 2 is a different wave-bearing structure. But, one can define and derive the analogous admittance matrix for the pair of fluid-loaded plates in the absence of the internal attachment. An explicit solution has recently been given by the authors,[12] and it is a full matrix, with coupling between force and rotation and between moment and deflection. The solution to the simpler but nontrivial scattering problem for the dissimilar plate junction without the internal attachment is also required, but this too has been recently solved, by Norris and Wickham.[13] Together, the analyses of Norris and Rebinsky[12] and Norris and Wickham[13] provide the ingredients for the solution to the three-member scattering problem. In the limit that plates 1 and 2 are identical then the Norris and Wickham[13] solution is trivial, and all that is required is the diagonal admittance matrix.[3,11]

In Sec. I we formulate the dynamic equations used to model the plates and internal frame, see Fig. 1. The plates are described by the classical theory of flexure and the reinforcement is assumed to be adequately characterized by a two-degree-of-freedom attachment. The formal solution is outlined in Sec. II where the whole result is split into three components. The first part is the specularly reflected field for

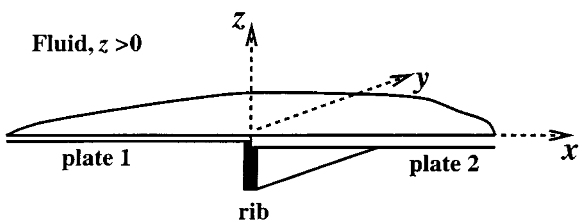

FIG. 1. The three-member junction and coordinate system.

a plate of infinite extent composed entirely of plate 1 and is relatively simple. The remaining two components rely on previous results concerning two joined dissimilar plates[13] and their corresponding line admittance matrix.[12] The detailed, explicit form of the solution is presented in Sec. III, where several limiting cases of interest are discussed. Finally in Sec. IV, we discuss and show the various diffraction coefficients of the wave fields emanating from the junction. We close with illustrations of the reflected, transmitted, and radiated energies of the scattered wave fields.

I. DEFINITION OF THE PROBLEM

We consider time harmonic motion of frequency $\omega>0$, with the term $e^{-i\omega t}$ understood but suppressed. The fluid-loaded plates lie in the plane $z=0$ and meet along $x=0$, $-\infty<y<\infty$, see Fig. 1. They are semi-infinite and uniform but can differ in density, elastic properties, and thickness. The dynamic behavior of each plate is modeled by the classical theory of flexure. Thus

$$B_j\nabla^2\nabla^2 w(x,y)-m_j\omega^2 w(x,y)=-p(x,y,0), \tag{1}$$

where $w(x,y)$ is the plate deflection in the z direction, and $p(x,y,z)$ is the acoustic pressure in the fluid, which occupies the half space $0<z<\infty$. Also, $m_{1,2}$ are the areal mass densities, and $B_{1,2}$ the bending stiffnesses of the distinct plates, and $j=1$ or 2 for $x<0$ and $x>0$, respectively. The relevant bending moment and effective shear force on either plate are given by the classical relations

$$M(x,y)=-B_j[w_{,xx}(x,y)+\nu_j w_{,yy}(x,y)], \tag{2a}$$

$$V(x,y)=M_{,x}-2B_j(1-\nu_j)w_{,xyy}(x,y), \tag{2b}$$

where ν_j is Poisson's ratio and $j=1$ and 2 for $x<0$ and >0, respectively. The complex-valued acoustic pressure satisfies the Helmholtz equation in the fluid, with wave number $k=\omega/c$, where c is the fluid sound speed. Finally, the pressure and deflection are related by the continuity condition

$$\rho\omega^2 w(x,y)=\frac{\partial p}{\partial z}(x,y,0), \quad -\infty<x,y<\infty, \tag{3}$$

where ρ is the fluid mass density per unit volume.

The phase factor $e^{ik_y y}$ is assumed for all dynamic quantities, and is explicitly removed thereby suppressing the y dependence. Thus we define $\bar{p}$ and $\bar{w}$ by

$$p(x,y,z)=\bar{p}(x,z)e^{ik_y y}, \quad w(x,y)=\bar{w}(x)e^{ik_y y}, \tag{4}$$

with analogous definitions for $\bar{M}(x)$ and $\bar{V}(x)$, and the Helmholtz equation becomes

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+\bar{k}^2\right)\bar{p}=0, \quad -\infty<x<\infty, \quad 0<z<\infty, \tag{5}$$

where $\bar{k}$ is the effective wave number, defined by $\bar{k}^2=k^2-k_y^2$. At the same time, elimination of the displacement $w(x,y)$ between the two boundary conditions (1) and (3) gives a single equation for the pressure on each half-plane (now line),

$$\mathscr{L}_1\bar{p}(x,0)=0, \quad x<0; \quad \mathscr{L}_2\bar{p}(x,0)=0, \quad x>0, \tag{6}$$

with the operators defined as

$$\mathscr{L}_j\equiv 1+a_j\left[\kappa_j^{-4}\left(\frac{\partial^2}{\partial x^2}-k_y^2\right)^2-1\right]\frac{\partial}{\partial z}, \quad j=1,2, \tag{7}$$

where $\kappa_{1,2}$ are the flexural wave numbers of the plates, and $a_{1,2}$ are the "null frequency" lengths,

$$\kappa_j^4=\omega^2 m_j/B_j, \quad a_j=m_j/\rho, \quad j=1,2. \tag{8}$$

The null frequency at which $ka=1$ $(m\omega=\rho c)$ provides a possible criterion for defining the transition from low- to high-frequency regimes for each fluid-loaded plate.

Suppose that an internal frame is attached at the junction of the two dissimilar plates located along $x=0$. Concerning the connection of all the components (plates and internal frame), coupling of out-of-plane to in-plane motion occurs because of the mismatch of neutral axes. To leading order, this coupling is assumed to be negligible and we have ignored it. Or from a different point of view, an academic problem has been posed where we have chosen the neutral axes of the plates and that of the internal frame to coincide.

For simplicity, we characterize the internal frame as a two degree of freedom attachment, with displacement $w_F(t)$ and rotation $\phi_F(t)$. Its motion induces a reaction force F_F and torque T_F on the junction. The kinematics of the three member junction imply that $w_F=\bar{w}(0)$ and $\phi_F=-\bar{w}'(0)$. We define a rib impedance such that

$$\begin{bmatrix} F_F \\ T_F \end{bmatrix}=-\overline{\mathbf{Z}}^{(\mathrm{rib})}\begin{bmatrix} -i\omega w_F \\ -i\omega\phi_F \end{bmatrix}. \tag{9}$$

For example, if m_F, j_F are the frame mass per unit length and rotary inertia per unit length, respectively, then the impedance is diagonal,

$$\overline{\mathbf{Z}}^{(\mathrm{rib})}=\begin{bmatrix} -i\omega m_F & 0 \\ 0 & -i\omega j_F \end{bmatrix}. \tag{10}$$

We note that the following analysis is not limited to this simple rib model, but can be applied to nonlocal impedances for wave-bearing internals.

The internal frame can be replaced by an equivalent force and moment loading along the junction of the two fluid-loaded plates. Equations (6) hold for all nonzero values of x, but not at $x=0$, where certain jump conditions need to be imposed. At $x=0$, the internal frame induces on the plate system a phased-line force in the positive z direction, $F_F e^{ik_y y}$, and a phased-line moment about the y axis in the clockwise direction, $T_F e^{ik_y y}$, such that

$$\bar{M}(0+)-\bar{M}(0-)=-T_F, \tag{11a}$$

$$\bar{V}(0+)-\bar{V}(0-)=-F_F. \tag{11b}$$

The reduced moment and shear force follow from (2) as

$$\bar{M}(x) = -B[\bar{w}_{,xx}(x) - \nu k_y^2 \bar{w}(x)], \tag{12a}$$

$$\bar{V}(x) = -B[\bar{w}_{,xxx}(x) - (2-\nu) k_y^2 \bar{w}_{,x}(x)], \tag{12b}$$

with the appropriate values taken for B and ν depending as x is positive or negative.

The problem is therefore, that the pressure $\bar{p}$ satisfies the Helmholtz equation (5) and the boundary conditions (6) on the plates. The deflection $\bar{w}$ and its first derivative $\bar{w}' = d\bar{w}/dx$ are both continuous at the junction, where $\bar{w}$ is related to $\bar{p}$ by (3). In addition, the junction conditions (9), (11), and (12) must hold. Finally, the scattered wave fields in the fluid and on the plates must satisfy the radiation condition as $\sqrt{x^2+z^2} \to \infty$.

II. FORMULATION OF THE GENERAL SOLUTION

A. Formal solution

We first write the total solution as the sum of three components,

$$\bar{p}(x,z) = \bar{p}_a(x,z) + \bar{p}_b(x,z) + \bar{p}_c(x,z),$$
$$\bar{w}(x) = \bar{w}_a(x) + \bar{w}_b(x) + \bar{w}_c(x). \tag{13}$$

Similarly, we split the moment and force as $\bar{M} = \bar{M}_a + \bar{M}_b + \bar{M}_c$ and $\bar{V} = \bar{V}_a + \bar{V}_b + \bar{V}_c$, respectively. Each separate solution has an applied load at the junction of force F_α, and moment T_α, for $\alpha = a$, b, and c. Thus

$$\begin{aligned} &\bar{M}_\alpha(0+) - \bar{M}_\alpha(0-) = -T_\alpha, \\ &\bar{V}_\alpha(0+) - \bar{V}_\alpha(0-) = -F_\alpha, \quad \alpha = a,b, \quad \text{and } c. \end{aligned} \tag{14}$$

We choose the solutions so that they each satisfy the Helmholtz equation (5) and the first of the two boundary conditions of Eq. (6). The second condition of Eq. (6) and the junction conditions at $x=0$ are in general, only satisfied by the total solution. In this regard, we note that the conditions (11) may be replaced by the equivalent pair

$$T_a + T_b + T_c = T_F, \quad F_a + F_b + F_c = F_F. \tag{15}$$

B. Solution a

The pressure $\bar{p}_a$ and displacement $\bar{w}_a$ are chosen as the incident wave with horizontal wave number ξ_0 (the y component has been suppressed for convenience) which satisfies the boundary condition on $x<0$. It is assumed to be one of the following:

$$\bar{p}_a(x,z) = e^{i\xi_0 x} \times \begin{cases} e^{-\gamma(\xi_0) z}, & \text{plate wave}, \\ [e^{\gamma(\xi_0) z} + \mathscr{R}_1(\xi_0) e^{-\gamma(\xi_0) z}], & \text{acoustic wave}. \end{cases} \tag{16}$$

The plate wave number ξ_0 is the root of $D_1(\xi)=0$ [see (18) below] which exists at all frequencies, and corresponds to the subsonic flexural wave. The square root $\gamma(\xi) = (\xi^2 - \bar{k}^2)^{1/2}$ is defined as an analytic function in the complex ξ plane cut so that its real part is non-negative. Along the real axis, $\gamma(\xi) = -i\sqrt{k^2-\xi^2}$ for $|\xi| < \bar{k}$ and $\gamma(\xi) = \sqrt{\xi^2 - k^2}$ for $|\xi| > \bar{k}$. We have selected this branch for γ so that the Fourier superpositions of solutions are outgoing at infinity. Also, $\mathscr{R}_1$ is the plane wave reflection coefficient for plate 1. Thus

$$\mathscr{R}_j(\xi) = 1 - 2/D_j(\xi), \quad j = 1 \text{ or } 2, \tag{17}$$

where

$$D_j(\xi) = 1 - a_j \gamma(\xi) (\kappa_j^{-4} (\xi^2 + k_y^2)^2 - 1). \tag{18}$$

Numerical calculations involving the branch cuts and the choice of roots are simplified by giving $\bar{k}$ a small imaginary part, i.e., $\bar{k} = |\bar{k}| e^{i\epsilon}$, $0 < \epsilon \ll 1$. This is consistent with the physical restrictions imposed by the radiation condition, and guarantees the existence of a strip of analyticity for certain functions. The strip is defined by $\xi \in \mathscr{H}^+ \cap \mathscr{H}^-$, where $\mathscr{H}^\pm$ are the upper and lower halves of the complex ξ plane. It is assumed that $D_j(\xi) \neq 0$, $\xi \in \mathscr{H}^+ \cap \mathscr{H}^-$, and that $\xi_0 \in \mathscr{H}^+$.

C. Solution b

Using (13) with (16), the plate Eq. (6) become

$$\mathscr{L}_1[\bar{p}_b(x,0) + \bar{p}_c(x,0)] = 0, \quad x<0, \tag{19a}$$

$$\mathscr{L}_2[\bar{p}_b(x,0) + \bar{p}_c(x,0)] = -\mathscr{L}_2 \bar{p}_a(x,0), \quad x>0. \tag{19b}$$

We choose the pressure $\bar{p}_b$ such that the contribution from $\bar{p}_a$ in the right member of Eq. (19b) is cancelled. This is accomplished by writing $\bar{p}_b$ as a Fourier integral of the form,[13]

$$\bar{p}_b(x,z) = -A_0 \bar{p}_0(x,z), \tag{20}$$

where

$$\bar{p}_0(x,z) \equiv \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{G(\xi_0)}{G(\xi)} \frac{e^{[i\xi x - \gamma(\xi) z]}}{\xi - \xi_0} d\xi, \tag{21}$$

and

$$A_0 = \begin{cases} 1, & \text{plate wave}, \\ \mathscr{R}_1(\xi_0) - \mathscr{R}_2(\xi_0), & \text{acoustic wave}. \end{cases} \tag{22}$$

We also define the generalized dispersion function

$$G(\xi) \equiv D_2(\xi)/K^+(\xi) = D_1(\xi)/K^-(\xi), \tag{23}$$

where $K^\pm(\xi)$ are the unique Wiener–Hopf factors of the quotient function

$$K(\xi) = D_1(\xi)/D_2(\xi), \tag{24}$$

such that

$$K(\xi) = K^-(\xi)/K^+(\xi), \quad K^-(-\xi) = 1/K^+(\xi). \tag{25}$$

Thus by definition, $K^\pm(\xi)$ are analytic in the half-planes $\mathscr{H}^\pm$. An explicit formula for $K^+(\xi)$ is given in the Appendix, based upon a factorization method developed by Norris and Wickham.[13] The form of the pressure $\bar{p}_b$ in (20) follows directly from some results concerning the acoustic diffraction from two joined flat plates.[13] It may easily be verified by direct substitution of $\bar{p}_b$ defined by Eqs. (20) through (24) that it satisfies

$$\mathscr{L}_1 \bar{p}_b(x,0) = 0, \quad x<0, \tag{26a}$$

$$\mathscr{L}_2 \bar{p}_b(x,0) = -\mathscr{L}_2 \bar{p}_a(x,0), \quad x>0, \tag{26b}$$

as claimed.

D. Solution c

The plate equation (19) are now reduced to homogeneous equations for $\bar{p}_c$,

$$\mathscr{L}_1\bar{p}_c(x,0)=0, \quad x<0; \quad \mathscr{L}_2\bar{p}_c(x,0)=0, \quad x>0. \tag{27}$$

The solution is apparently $\bar{p}_c\equiv 0$, however, a nonzero applied force F_c and a moment T_c are required at $x=0$ in order to satisfy the junction conditions there. The $\bar{p}_c$ solution is therefore analogous to that which describes the line admittance at the junction of two plates, as discussed in Ref. 12. We first write the $\bar{p}_c$ as a Fourier integral of the form

$$\bar{p}_c(x,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{p}_c(\xi)e^{[i\xi x-\gamma(\xi)z]}\,d\xi. \tag{28}$$

The general solution of the homogeneous dual integral equations (27) which satisfies the kinematic continuity conditions is then[13]

$$\tilde{p}_c(\xi)=(\bar{A}_0+\bar{A}_1\xi)/G(\xi), \tag{29}$$

where, following the analysis of Norris and Rebinsky,[12] the coefficients $\bar{A}_0$ and $\bar{A}_1$ can be linearly related to the unknown loads at the junction,

$$\begin{bmatrix}\bar{A}_1\\ \bar{A}_0\end{bmatrix}=-2i(\mathbf{N}_1+\mathbf{N}_2)^{-1}\begin{bmatrix}-T_c\\ iF_c\end{bmatrix}. \tag{30}$$

Therefore, using Eqs. (28) through (30) the additional pressure is

$$\bar{p}_c(x,z)=\frac{1}{i\pi}\int_{-\infty}^{\infty}(\xi,1)(\mathbf{N}_1+\mathbf{N}_2)^{-1}\times\begin{bmatrix}-T_c\\ iF_c\end{bmatrix}\frac{e^{[i\xi x-\gamma(\xi)z]}}{G(\xi)}\,d\xi. \tag{31}$$

The derivation of Eq. (30) is lengthy, but is very similar to the derivation of the admittance matrix for the fluid-loaded system of joined plates, and we refer the reader to Norris and Rebinsky[12] for details. For our purposes, all that is required are the matrices $\mathbf{N}_1$ and $\mathbf{N}_2$, which are defined as

$$\mathbf{N}_n(\vartheta)=\begin{bmatrix}\left(1-\frac{\vartheta k_y^2}{k_y^2+\zeta_n^2}\right)\cosh\ \sigma_n & \left(1-\frac{\vartheta k_y^2}{k_y^2+\zeta_n^2}\right)\zeta_n^{-1}\ \sinh\ \sigma_n\\ \left(1+\frac{\vartheta k_y^2}{k_y^2+\zeta_n^2}\right)\zeta_n\ \sinh\ \sigma_n & \left(1+\frac{\vartheta k_y^2}{k_y^2+\zeta_n^2}\right)\cosh\ \sigma_n\end{bmatrix},\quad n=1\ \text{or}\ 2, \tag{32}$$

and the number ϑ is an averaged difference in the material properties of the two plates

$$\vartheta\equiv 1-(\nu_2B_2-\nu_1B_1)/(B_2-B_1). \tag{33}$$

Also, $\pm\zeta_1\in\mathscr{H}^{\pm}$ and $\pm\zeta_2\in\mathscr{H}^{\pm}$ are the roots of $\mathscr{R}_2(\xi)=\mathscr{R}_1(\xi)$, or

$$\zeta_1^2+\zeta_2^2=-2k_y^2, \quad \zeta_1^2\zeta_2^2=k_y^4-\zeta_0^4, \tag{34}$$

where

$$\zeta_0^4=\omega^2\Delta m/\Delta B. \tag{35}$$

Many of the subsequent equations are simplified by definite choices for the roots. We therefore choose them as

$$\zeta_1^2=\zeta_0^2-k_y^2, \quad \zeta_2^2=-\zeta_0^2-k_y^2. \tag{36}$$

The roots depend on the wave numbers ζ_0 and k_y, and $\zeta_1\neq\zeta_2$ as long as $\zeta_0^4\neq 0$. We assume this to be the case, for simplicity. Finally, σ_1 and σ_2 are defined by

$$\sigma_n=\log\ K^+(\zeta_n), \quad n=1\ \text{or}\ 2. \tag{37}$$

E. The total solution

We now combine $\bar{p}_a$ of Eq. (16), $\bar{p}_b$ of Eqs. (20) and (21), and $\bar{p}_c$ of Eqs. (28) through (30), to obtain, using Eq. (13),

$$\begin{aligned}\bar{p}(x,z)&=\bar{p}_a(x,z)\\ &\quad-\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{A(\xi)}{\xi-\xi_0}\frac{G(\xi_0)}{G(\xi)}e^{[i\xi x-\gamma(\xi)z]}\,d\xi\\ &=\bar{p}_a(x,z)-A\left(-i\frac{\partial}{\partial x}\right)\bar{p}_0(x,z),\end{aligned} \tag{38}$$

where $\bar{p}_0(x,z)$ is defined in Eq. (21), and A is a quadratic polynomial,

$$A(\xi)=A_0-\frac{2(\xi-\xi_0)}{G(\xi_0)}(\xi,1)(\mathbf{N}_1+\mathbf{N}_2)^{-1}\begin{bmatrix}-T_c\\ iF_c\end{bmatrix}. \tag{39}$$

The solution defined by Eqs. (38) and (39) is similar to that obtained by Norris and Wickham[13] for the scattered pressure from two joined flat plates. Note that the three member problem is now reduced to finding T_c and F_c which define the polynomial $A(\xi)$.

III. EXPLICIT SOLUTION AND SPECIAL LIMITS

A. Applied moment and force loading

The general solution depends upon the line loads T_c and F_c, which represent the combined contribution of the frame moment T_F and force F_F plus a component from the moment and force jumps T_a+T_b and F_a+F_b generated at the junction. The latter are defined by Eq. (14), and they occur because the solution $\bar{p}_a+\bar{p}_b$ does not completely satisfy the continuity conditions at $x=0$ for a two plate system. The total force and moment balance at the junction requires that (15) hold, or

$$\begin{bmatrix} -T_c \\ iF_c \end{bmatrix} = \begin{bmatrix} -T_F \\ iF_F \end{bmatrix} - \begin{bmatrix} -T_a - T_b \\ iF_a + iF_b \end{bmatrix}. \tag{40}$$

Also, the unknowns T_c and F_c can be expressed as

$$\begin{bmatrix} -T_c \\ iF_c \end{bmatrix} = \mathbf{Z} \begin{bmatrix} \bar{w}_c(0) \\ -i\bar{w}_c'(0) \end{bmatrix}, \tag{41}$$

where $\mathbf{Z}$ is the impedance matrix for two joined flat plates under fluid loading.[12] It is given explicitly below in (55). Note that the vectors in Eq. (41) differ from those in, for instance, Eq. (9). Based upon the analysis in Ref. 12 we find it more convenient to redefine the impedance matrices so that they relate vectors as in (41). This definition of the impedance is unconventional, but it can be easily reconciled with standard procedure.[12] We therefore redefine the rib impedance matrix to be consistent with this new definition; thus

$$\begin{bmatrix} -T_F \\ iF_F \end{bmatrix} = -\mathbf{Z}^{(\mathrm{rib})} \begin{bmatrix} \bar{w}(0) \\ -i\bar{w}'(0) \end{bmatrix}. \tag{42}$$

Comparing (9) and (42), we have

$$\begin{bmatrix} Z_{11}^{(\mathrm{rib})} & Z_{12}^{(\mathrm{rib})} \\ Z_{21}^{(\mathrm{rib})} & Z_{22}^{(\mathrm{rib})} \end{bmatrix} = \omega \begin{bmatrix} i\bar{Z}_{21}^{(\mathrm{rib})} & \bar{Z}_{22}^{(\mathrm{rib})} \\ \bar{Z}_{11}^{(\mathrm{rib})} & -i\bar{Z}_{12}^{(\mathrm{rib})} \end{bmatrix}. \tag{43}$$

Combining Eqs. (40) through (42) with the second of (13), we find that

$$\begin{bmatrix} -T_c \\ iF_c \end{bmatrix} = -\mathbf{Z}(\mathbf{Z}+\mathbf{Z}^{(\mathrm{rib})})^{-1} \left\{ \begin{bmatrix} -T_a - T_b \\ iF_a + iF_b \end{bmatrix} + \mathbf{Z}^{(\mathrm{rib})} \begin{bmatrix} \bar{w}_a(0) + \bar{w}_b(0) \\ -i\bar{w}_a'(0) - i\bar{w}_b'(0) \end{bmatrix} \right\}. \tag{44}$$

The jumps $T_a + T_b$ and $F_a + F_b$ follow from Eqs. (12) and (14) as

$$T_a + T_b = (B_2 - B_1)[(\bar{w}_a + \bar{w}_b)_{,xx}(0) - (1-\vartheta) \times k_y^2 (\bar{w}_a + \bar{w}_b)(0)], \tag{45a}$$

$$F_a + F_b = (B_2 - B_1)[(\bar{w}_a + \bar{w}_b)_{,xxx}(0) - (1+\vartheta) \times k_y^2 (\bar{w}_a + \bar{w}_b)_{,x}(0)]. \tag{45b}$$

The displacement $\bar{w}_a + \bar{w}_b$ can be found fairly readily from the analyses of Norris and Wickham[13] and Norris and Rebinsky.[12] Thus equation (4.13) of Ref. 13 with $A_n = A_0 \delta_{n0}$, and equation (B.11) of Ref. 13 with appropriate modification of the terms $u_m^{\pm}$ for oblique incidence, as given in Eq. (59) of Ref. 12, together imply that

$$(B_2 - B_1)(\bar{w}_a + \bar{w}_b) = -A_0 \sum_{n=0}^{3} \lambda_n \frac{(ix)^n}{n!} + O(x^4), \tag{46}$$

where

$$\lambda_n = G(\xi_0) \sum_{m=1}^{2} \frac{\zeta_m^{n-1}}{4(\zeta_m^2 + k_y^2)} \times \left(\frac{e^{\sigma_m}}{\zeta_m - \xi_0} + (-1)^n \frac{e^{-\sigma_m}}{\zeta_m + \xi_0} \right). \tag{47}$$

Note that $\bar{w}_a + \bar{w}_b$ and its first three derivatives are continuous at $x=0$.

It follows from Eqs. (45) and (46) that

$$\begin{bmatrix} -T_a - T_b \\ iF_a + iF_b \end{bmatrix} = -A_0 \begin{bmatrix} \lambda_2 + (1-\vartheta) k_y^2 \lambda_0 \\ \lambda_3 + (1+\vartheta) k_y^2 \lambda_1 \end{bmatrix}. \tag{48}$$

After substituting for the λ_n's using (47) and some straightforward algebraic manipulations, Eq. (48) can be written simply as

$$\begin{bmatrix} -T_a - T_b \\ iF_a + iF_b \end{bmatrix} = -\frac{1}{2} A_0 G(\xi_0) [(\zeta_1^2 - \xi_0^2)^{-1} \mathbf{N}_1 + (\zeta_2^2 - \xi_0^2)^{-1} \mathbf{N}_2] \begin{bmatrix} 1 \\ \xi_0 \end{bmatrix}. \tag{49}$$

Similarly,

$$\begin{bmatrix} \bar{w}_a(0) + \bar{w}_b(0) \\ -i\bar{w}_a'(0) - i\bar{w}_b'(0) \end{bmatrix} = -\frac{A_0 G(\xi_0)}{2\zeta_0^2 (B_2 - B_1)} [(\zeta_1^2 - \xi_0^2)^{-1} \mathbf{M}_1 - (\zeta_2^2 - \xi_0^2)^{-1} \mathbf{M}_2] \begin{bmatrix} 1 \\ \xi_0 \end{bmatrix}, \tag{50}$$

where

$$\mathbf{M}_n^{\pm 1} = \begin{bmatrix} \cosh \sigma_n & \pm \zeta_n^{-1} \sinh \sigma_n \\ \pm \zeta_n \sinh \sigma_n & \cosh \sigma_n \end{bmatrix}, \quad n = 1 \text{ or } 2. \tag{51}$$

Also, the matrices $\mathbf{N}_n$ and $\mathbf{M}_n$ are related through the expression

$$\mathbf{N}_n(\vartheta) = \mathbf{J}_n(\vartheta) \mathbf{M}_n, \tag{52}$$

where (Ref. 12)

$$\mathbf{J}_n(\vartheta) = \begin{bmatrix} 1 - \dfrac{\vartheta k_y^2}{k_y^2 + \zeta_n^2} & 0 \\ 0 & 1 + \dfrac{\vartheta k_y^2}{k_y^2 + \zeta_n^2} \end{bmatrix}. \tag{53}$$

Upon substitution of (49) and (50) into (44), we obtain the desired loads for problem c as

$$\begin{bmatrix} -T_c \\ iF_c \end{bmatrix} = \frac{A_0}{2} G(\xi_0) \mathbf{Z}(\mathbf{Z}+\mathbf{Z}^{(\mathrm{rib})})^{-1} \{ (\zeta_1^2 - \xi_0^2)^{-1} \times [\mathbf{J}_1(\vartheta) + \zeta_0^{-2} (B_2 - B_1)^{-1} \mathbf{Z}^{(\mathrm{rib})}] \mathbf{M}_1 + (\zeta_2^2 - \xi_0^2)^{-1} \times [\mathbf{J}_2(\vartheta) - \zeta_0^{-2} (B_2 - B_1)^{-1} \mathbf{Z}^{(\mathrm{rib})}] \mathbf{M}_2 \} \begin{bmatrix} 1 \\ \xi_0 \end{bmatrix}, \tag{54}$$

where the junction impedance matrix $\mathbf{Z}$ can be written as[12]

$$\mathbf{Z} = \zeta_0^2 (B_2 - B_1)(\mathbf{N}_1 + \mathbf{N}_2)(\mathbf{M}_1 - \mathbf{M}_2)^{-1}. \tag{55}$$

Hence, the polynomial $A(\xi)$ using Eq. (39) is given by

$$\frac{A(\xi)}{A_0}=1-(\xi-\xi_0)(\xi,1)(\mathbf{M}_1-\mathbf{M}_2)^{-1}(\mathbf{Z}+\mathbf{Z}^{(\mathrm{rib})})^{-1}$$
$$\times\{(\zeta_1^2-\xi_0^2)^{-1}[\zeta_0^2(B_2-B_1)\mathbf{J}_1(\vartheta)+\mathbf{Z}^{(\mathrm{rib})}]\mathbf{M}_1$$
$$+(\zeta_2^2-\xi_0^2)^{-1}[\zeta_0^2(B_2-B_1)\mathbf{J}_2(\vartheta)-\mathbf{Z}^{(\mathrm{rib})}]\mathbf{M}_2\}$$
$$\times\begin{bmatrix}1\\ \xi_0\end{bmatrix}. \tag{56}$$

This can be further simplified as

$$\frac{A(\xi)}{A_0}=1-(\xi-\xi_0)(\xi,1)(\mathbf{X}_1+\mathbf{X}_2)^{-1}\left[\frac{\mathbf{X}_1}{\zeta_1^2-\xi_0^2}+\frac{\mathbf{X}_2}{\zeta_2^2-\xi_0^2}\right]$$
$$\times\begin{bmatrix}1\\ \xi_0\end{bmatrix}, \tag{57}$$

where

$$\mathbf{X}_1=\zeta_0^2(B_2-B_1)\mathbf{N}_1+\mathbf{Z}^{(\mathrm{rib})}\mathbf{M}_1,$$
$$\mathbf{X}_2=\zeta_0^2(B_2-B_1)\mathbf{N}_2-\mathbf{Z}^{(\mathrm{rib})}\mathbf{M}_2. \tag{58}$$

We have now completed the general solution for the problem. Thus the function $A(\xi)$ of Eq. (57), combined with Eqs. (16), (21), and (38), provides a general and explicit formula for the pressure in the fluid scattered from the three member structure.

B. The limit of no rib, and of a clamped junction

It is of interest to examine two special limiting configurations, which are independent of the internal member. When $\mathbf{Z}^{(\mathrm{rib})}\to 0$, the limiting case of two fluid-loaded plates in welded contact is obtained, namely

$$\left.\frac{A(\xi)}{A_0}\right|_{\mathrm{welded}}=1-(\xi-\xi_0)(\xi,1)(\mathbf{N}_1+\mathbf{N}_2)^{-1}$$
$$\times\left[\frac{\mathbf{N}_1}{\zeta_1^2-\xi_0^2}+\frac{\mathbf{N}_2}{\zeta_2^2-\xi_0^2}\right]\begin{bmatrix}1\\ \xi_0\end{bmatrix}. \tag{59}$$

Conversely, as $\mathbf{Z}^{(\mathrm{rib})}\to\infty$, the limiting case of two clamped plates is obtained. Thus using Eq. (55) we find that

$$\left.\frac{A(\xi)}{A_0}\right|_{\mathrm{clamped}}=1-(\xi-\xi_0)(\xi,1)(\mathbf{M}_1-\mathbf{M}_2)^{-1}$$
$$\times\left[\frac{\mathbf{M}_1}{\zeta_1^2-\xi_0^2}-\frac{\mathbf{M}_2}{\zeta_2^2-\xi_0^2}\right]\begin{bmatrix}1\\ \xi_0\end{bmatrix}. \tag{60}$$

By setting $k_y=0$ in the above expressions one recovers the two-dimensional form of the polynomial A for both the welded and clamped cases, as derived by Norris and Wickham.[13]

C. A uniform plate

We may now consider the limit of two completely identical plates, i.e., a single uniform plate of infinite extent, with an attached internal frame. The "incident" pressure $\bar{p}_a$ remains the same, but now $\bar{p}_b$ is identically zero. Various other simplifications result from this limit. Thus $D_1(\xi)=D_2(\xi)\equiv D(\xi)$, $K^{\pm}\to 1$ and hence $G(\xi)=D(\xi)$, using Eq. (23). For simplicity, let $\Delta B\to 0$, keeping Δm finite. Then, $|\zeta_0|$, $|\zeta_1|$, and $|\zeta_2|\to\infty$, while $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{N}_1$, and $\mathbf{N}_2\to\mathbf{I}$. Thus the c pressure becomes, using Eq. (31),

$$\bar{p}_c=\frac{1}{2\pi}\int_{-\infty}^{\infty}(F_c+i\xi T_c)D^{-1}(\xi)e^{[i\xi x-\gamma(\xi)z]}\,d\xi. \tag{61}$$

The c-loads at the junction follow from Eq. (44) and the identities $T_a=T_b=F_a=F_b=\bar{w}_b(0)=\bar{w}_b'(0)=0$, as

$$\begin{bmatrix}-T_c\\ iF_c\end{bmatrix}=-\bar{w}_a(0)\mathbf{Z}(\mathbf{Z}+\mathbf{Z}^{(\mathrm{rib})})^{-1}\mathbf{Z}^{(\mathrm{rib})}\begin{bmatrix}1\\ \xi_0\end{bmatrix}, \tag{62}$$

and

$$\bar{w}_a(0)=\begin{cases}-\gamma(\xi_0)/\rho\omega^2, & \text{plate wave,}\\ \gamma(\xi_0)[1-\mathscr{R}(\xi_0)]/\rho\omega^2, & \text{acoustic wave.}\end{cases} \tag{63}$$

The impedance matrix $\mathbf{Z}$ given by (55) (cf. Norris and Rebinsky[12]) becomes in the limit of two identical plates

$$\mathbf{Z}=\begin{bmatrix}0 & \left(\frac{1}{\rho\omega^2}\frac{\partial\mu_3}{\partial a}\right)^{-1}\\ \left(\frac{1}{\rho\omega^2}\frac{\partial\mu_1}{\partial a}\right)^{-1} & 0\end{bmatrix},$$
$$\text{identical plates,} \tag{64}$$

and we have shown that[12]

$$\frac{1}{\rho\omega^2}\frac{\partial\mu_j}{\partial a}=\frac{1}{2\pi B}\sum_{n=1}^{5}(\xi_n)^{j-2}$$
$$\times\left[\frac{\pi+2s_n\theta_n}{4(\xi_n^2+k_y^2)+s_n\kappa^4/a\gamma^3(\xi_n)}\right],\quad j=1\ \text{and}\ 3, \tag{65}$$

where $\xi_1,...,\xi_5$ are the five zeros of $P(\xi)=D(\xi)\bar{D}(\xi)$ in $\mathscr{H}^+$ and $\bar{D}(\xi)=2-D(\xi)$, i.e., they solve

$$[(\xi^2+k_y^2)^2-\kappa^4]^2(\xi^2-\bar{k}^2)-\kappa^8/a^2=0. \tag{66}$$

The complex angles $\theta_1,...,\theta_5$, are defined in accordance with the Appendix as $\theta_n=\cos^{-1}(\xi_n/\bar{k})$ and $s_n=1$ or -1 depending as ξ_n is a zero of $D(\xi)$ or $\bar{D}(\xi)$, respectively. Thus $s_n=1-D(\xi_n)$.

Combining Eqs. (61), (62), and (64), we determine A for two identical plates with an internal frame as

$$A(\xi)|_{\mathrm{identical}}=\frac{\bar{w}_a(0)}{D(\xi_0)}(\xi-\xi_0)(\xi,1)(\mathbf{Y}+\mathbf{Y}^{(\mathrm{rib})})^{-1}\begin{bmatrix}1\\ \xi_0\end{bmatrix}, \tag{67}$$

where $\mathbf{Y}=\mathbf{Z}^{-1}$ and $\mathbf{Y}^{(\mathrm{rib})}=(\mathbf{Z}^{(\mathrm{rib})})^{-1}$ are the junction and rib admittances, respectively. The total solution for the uniform plate with an internal attachment then follows from Eqs. (38), (63), and (67) as

$$\bar{p}(x,z)=\bar{p}_a(x,z)-\bar{w}_a(0)\left(-i\frac{\partial}{\partial x},1\right)$$
$$\times(\mathbf{Y}+\mathbf{Y}^{(\mathrm{rib})})^{-1}\begin{bmatrix}1\\ \xi_0\end{bmatrix}p_{\mathrm{line}}(x,z), \tag{68}$$

and

$$p_{\text{line}}(x,z)=\frac{1}{2\pi i}\int_{-\infty}^{\infty}D^{-1}(\xi)e^{[i\xi x-\gamma(\xi)z]}d\xi. \tag{69}$$

The pressure p_{line} corresponds to a unit line force applied on a uniform plate. Equation (68) can be expanded by using Eqs. (10), (43), and the off-diagonal form of **Z** in Eq. (64), to give

$$\bar{p}(x,z)=\bar{p}_a(x,z)-\rho\omega^2\bar{w}_a(0)\left[\left(\frac{\partial\mu_1}{\partial a}+\frac{i\rho}{m_F}\right)^{-1}\right.$$
$$\left.-i\xi_0\left(\frac{\partial\mu_3}{\partial a}+\frac{i\rho}{j_F}\right)^{-1}\frac{\partial}{\partial x}\right]p_{\text{line}}(x,z). \tag{70}$$

This provides a relatively simple formula for the response from an arbitrary rib to oblique incidence on a uniform plate.

IV. EXAMPLES

In all of the results shown here the material is the same for the entire structure. The contrast at the junction is provided by a discontinuity in plate thickness with three varying degrees of rib impedance: (a) infinite, (b) zero, and (c) finite. When the rib impedance is infinite, the structure corresponds to two plates clamped or fixed along the junction line. For zero rib impedance, the internal frame disappears and the structure behaves as two plates in welded contact. The numerical results are all for the material combination of steel and water, with a thickness change of 100% from the left plate to the right. This corresponds to $\alpha=2$, where α is the thickness ratio,

$$\alpha=h_2/h_1. \tag{71}$$

We assume a steel rib of rectangular cross section of thickness $h_R=0.5$ in. and length $l_R=4.5$ in. with mass per unit length m_F and rotary inertia per unit length $j_F=m_F(h_R^2/12+l_R^2/3)$ [cf. Eq. (10)]. The frequency dependence is discussed in terms of the nondimensional frequency Ω, normalized with respect to the coincidence frequency ω_{c1} of plate 1,

$$\Omega=\frac{\omega}{\omega_{c1}}\equiv\frac{k^2}{\kappa_1^2}. \tag{72}$$

Equation (71) implies that $\kappa_2=\kappa_1/\sqrt{\alpha}$, and hence the coincidence frequency of plate 2 is at $\Omega=1/\alpha=0.5$. Finally, all the results shown concern a subsonic flexural wave incident from $x<0$ (plate 1).

A. Diffraction coefficients

We assume that the observation distances from the junction are sufficiently large that far-field approximations can be used. The acoustic response follows by applying the method of steepest descent to the integral (38), while the structural response depends upon contributions from the poles associated with the subsonic flexural waves. All these diffracted waves can be characterized by junction diffraction coefficients. We refer the reader to the paper of Norris and Wickham[13] for a more detailed discussion of these coefficients and their reciprocal identities.

We first consider the scattered acoustic far-field pressure in the fluid, which can be written in terms of the diffraction coefficient $\mathscr{C}(\theta)$,

$$\bar{p}^s=\mathscr{C}(\theta)\sqrt{\frac{2}{\pi kr}}e^{-i\pi/4}e^{i\bar{k}\bar{r}},\quad \bar{k}\bar{r}\to\infty, \tag{73}$$

where

$$\mathscr{C}(\theta)=-\frac{1}{2}\gamma(\xi)\frac{G(\xi_0)}{G(\xi)}\frac{A(\xi)}{\xi-\xi_0}. \tag{74}$$

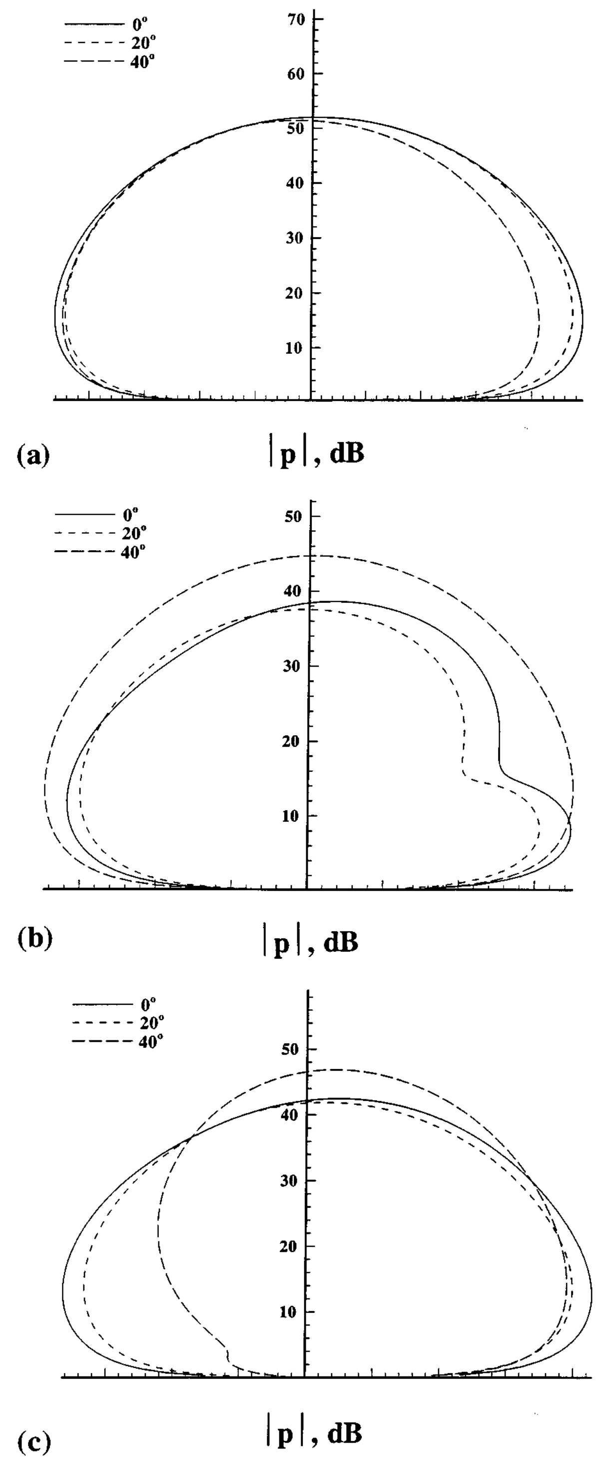

FIG. 2. Polar plots of the scattered acoustic pressure amplitude from the three member junction for normal incidence (0°) and two oblique angles of incidence at a frequency of $\Omega=0.3$. (a) infinite rib impedance, (b) zero rib impedance, and (c) finite rib impedance.

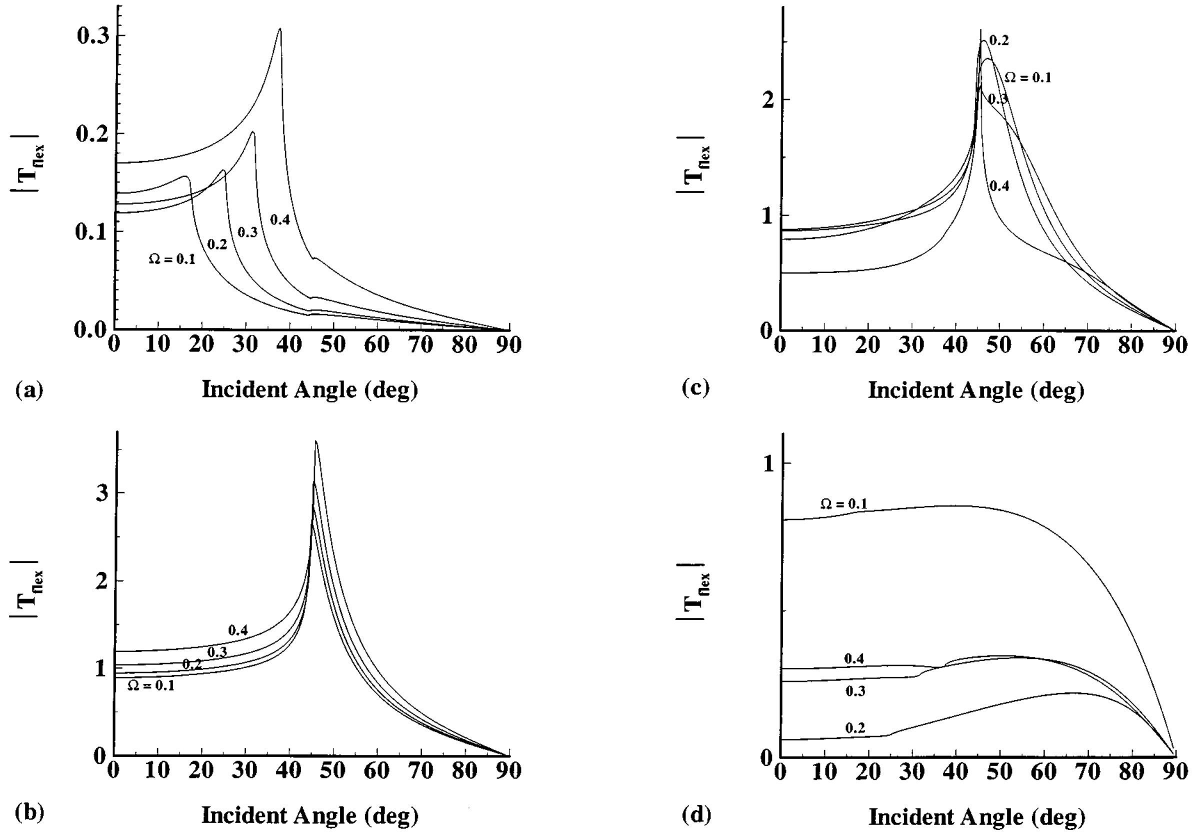

FIG. 3. The transmission coefficient as a function of oblique angle of incidence and frequency. (a) infinite rib impedance, (b) zero rib impedance, (c) finite rib impedance, and (d) same finite rib impedance as in (c) but plate 2 is half the thickness of plate 1.

The scattering angle θ is defined by $\xi = \bar{k}\cos\theta$, that is, it measures the angle between the positive x axis and the projection of the observation direction on the x-z plane. In Fig. 2, we show the directivity of acoustic diffraction for a subsonic wave launched on plate 1 striking the joint at different angles of incidence φ relative to normal, and for various rib impedances: (a) infinite or clamped plates, (b) zero or welded plates, and (c) finite. The curves in Fig. 2 indicate that at low frequencies, e.g., $\Omega=0.3$, the total radiated power is smallest for two welded dissimilar plates and largest when they are clamped. The response for a finite rib impedance lies between the welded and clamped results.

Next, we turn to the scattered subsonic flexural waves. We define the structural diffraction coefficients in terms of the scattered on-surface pressure on either plate far from the junction,

$$\begin{aligned} \bar{p}^s &= \mathscr{R}_{\text{flex}} e^{[i\xi_0^{(1)}x - \gamma(\xi_0^{(1)})z]}, \quad x<0; \\ \bar{p}^s &= \mathscr{T}_{\text{flex}} e^{[i\xi_0^{(2)}x - \gamma(\xi_0^{(2)})z]}, \quad x>0. \end{aligned} \tag{75}$$

Here $\xi_0^{(1)}$ and $\xi_0^{(2)}$ are the subsonic flexural wave numbers on plates $x<0$ and $x>0$, respectively, i.e., $D_1(\xi_0^{(1)})=0$ and $D_2(\xi_0^{(2)})=0$. The reflection and transmission coefficients are

$$\begin{aligned} \mathscr{R}_{\text{flex}} &= \frac{A(-\xi_0^{(1)})G(\xi_0^{(1)})}{2\xi_0^{(1)}K^+(\xi_0^{(1)})D_1'(\xi_0^{(1)})}, \\ \mathscr{T}_{\text{flex}} &= \frac{A(\xi_0^{(2)})G(\xi_0^{(1)})K^+(\xi_0^{(2)})}{(\xi_0^{(1)}-\xi_0^{(2)})D_2'(\xi_0^{(2)})}, \end{aligned} \tag{76}$$

respectively. Figure 3 shows the behavior of the transmission coefficient as a function of the angle of oblique incidence and of frequency. Again, the incident wave is a subsonic flexural wave on plate 1, and normal incidence corresponds to 0°. Several effects are evident from the curves. First, that the transmitted amplitude falls off rapidly at high angles of incidence, and secondly that the angle at which the transmission begins to diminish is an increasing function of frequency. Also, there appear to be different critical angles, and the angles depend upon the nature of the attachment.

Consider first the case of a rib of infinite impedance, Fig. 3(a). The transmission is apparently greatest at the critical angle defined by

$$\sin\ \varphi_f^{\text{crit}} = k/\kappa_1. \tag{77}$$

This angle relates the flexural wave number on plate 1 to the fluid wave number, and can be expressed using Eq. (72) as

$$\sin\ \varphi_f^{\text{crit}} = \sqrt{\Omega}. \tag{78}$$

The value of the critical angle therefore depends strongly upon frequency, which is evident from Fig. 3(a). For oblique angles of incidence beyond this critical value, the incident wave cannot travel through the adjacent fluid and transmission across the rigid rib is essentially suppressed. This phenomenon was previously illustrated by Lyapunov[10] and by Photiadis[11] for the case of a uniform plate with an infinite impedance line discontinuity. The results of Fig. 3(a) suggest that the effect is only weakly dependent on the properties of plate 2.

By contrast, in Fig. 3(b) we consider a rib of zero impedance. The same overall behavior is observed but a different critical angle is clearly operative, one which is independent of frequency. Physically, this critical angle describes the total internal reflection from a slow medium (thin plate) into a fast medium (thick plate) and here it relates the flexural wave number on plate 1 to that of plate 2,

$$\sin \ \varphi_p^{\rm crit} = \kappa_2/\kappa_1, \tag{79}$$

or, more specifically

$$\sin \ \varphi_p^{\rm crit} = 1/\sqrt{\alpha}, \qquad \text{for identical materials}, \tag{80}$$

which is exactly 45° for the pair of plates considered. It can also be observed to a lesser extent in Fig. 3(a) where it shows up as a kink in the curves after their maxima.

The case of a rib of finite impedance is shown in Fig. 3(c), and exhibits a combination of the effects seen in the two previous limiting cases. This is better observed by comparing differences in the transmitted energy which is discussed in the next section. But it is clear that the infinite impedance (rigid rib) does not represent the actual state of affairs with a finite impedance rib. The "plate" critical angle $\varphi_p^{\rm crit}$ associated with the zero impedance limit is more significant in this case. The energy results below will reinforce this conclusion. In Fig. 3(d) we illustrate the opposite plate configuration ($\alpha = \frac{1}{2}$) with an identical finite rib impedance to Fig. 3(c). For this case, the critical angle $\phi_p^{\rm crit}$ no longer is real (it is complex) and it does not play a role in the transmission. Thus, one will observe energy transmitted across the rib for all angles of incidence. But, the effect of the critical angle $\phi_f^{\rm crit}$ is observed as a ripple on the curves.

B. Energy redistribution

The flexural waves on each plate are both subsonic and provide the only means of energy transmission away from the junction, other than the acoustic radiation loss. Thus, assuming both flexural waves propagate, i.e., that $\varphi < \varphi_p^{\rm crit}$, then the statement of energy conservation[13] is

$$1 = |\mathscr{R}_{\rm flex}|^2 + \frac{D_2'(\xi_0^{(2)})\,\gamma(\xi_0^{(2)})}{D_1'(\xi_0^{(1)})\,\gamma(\xi_0^{(1)})}\,|\mathscr{T}_{\rm flex}|^2 - \frac{1}{D_1'(\xi_0^{(1)})\,\gamma(\xi_0^{(1)})}\,\frac{4}{\pi}\int_0^{\pi} |\mathscr{C}(\theta)|^2\, d\theta. \tag{81}$$

The three terms in the right member are each positive and less than unity, and correspond to the fractions of energy reflected on plate 1, transmitted on plate 2, and acoustically radiated into the fluid.

The separate components of reflected, transmitted, and diffracted energy are shown in Fig. 4. For two clamped dissimilar plates [Fig. 4(a)], most of the energy is carried by the reflected signal. There is a sharp transition at the "fluid" critical angle $\varphi_f^{\rm crit}$ of Eq. (78) where the transmitted and diffracted energies both become essentially zero. It is interesting to compare Fig. 4(a) with Fig. 3(a), which shows a significant transmitted pressure amplitude, but the associated energy in Fig. 4(a) is clearly small.

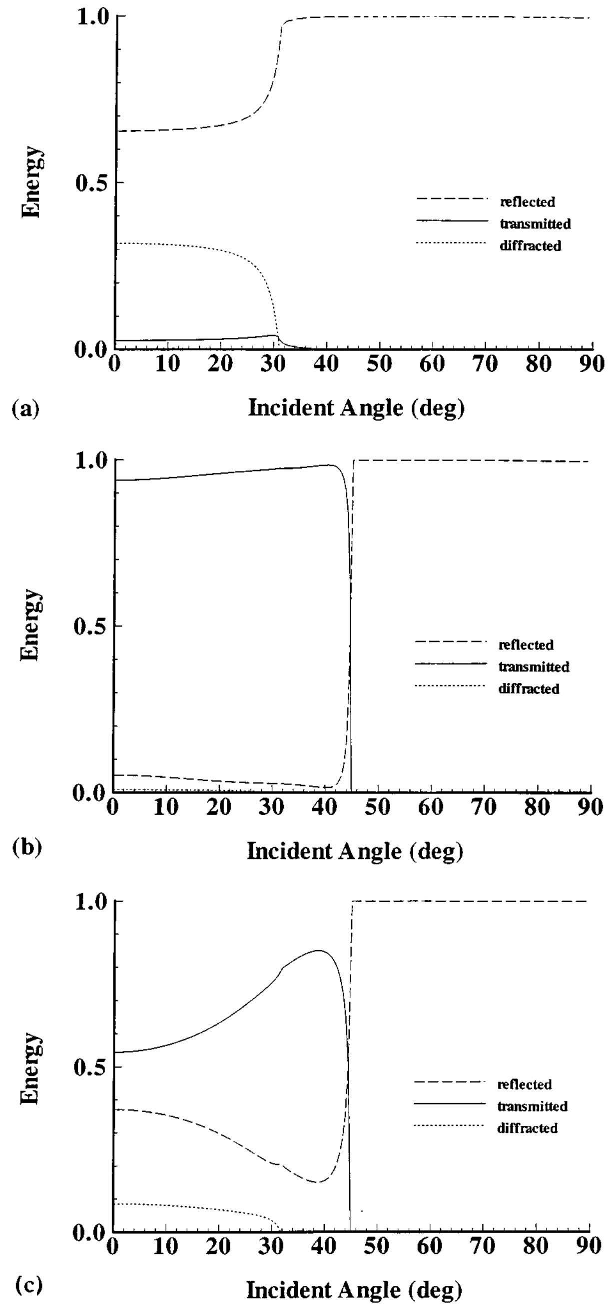

FIG. 4. The reflected, transmitted, and diffracted energy at a three-member junction as the oblique angle of incidence is varied at a frequency of $\Omega = 0.3$. (a) infinite rib impedance, (b) zero rib impedance, and (c) finite rib impedance.

In Fig. 4(b) the rib impedance is zero and almost all of the energy is transmitted for oblique angles of incidence less than the "plate" critical angle $\varphi_p^{\rm crit}$ given by Eq. (80). For angles above this value the energy is completely reflected. For a rib of finite impedance, Fig. 4(c), we again obtain a mixture of the two limiting cases. We have chosen the dimensions of the rib (i.e., impedance $\mathbf{Z}^{\rm rib}$) so that its transmission behavior will lie qualitatively midway in between that of zero and infinite. One can clearly see the simultaneous effects of both critical angles. Acoustic radiation ceases beyond the fluid critical angle $\varphi_f^{\rm crit}(\sim 31°)$, while the transmission of structural energy is totally suppressed at the plate

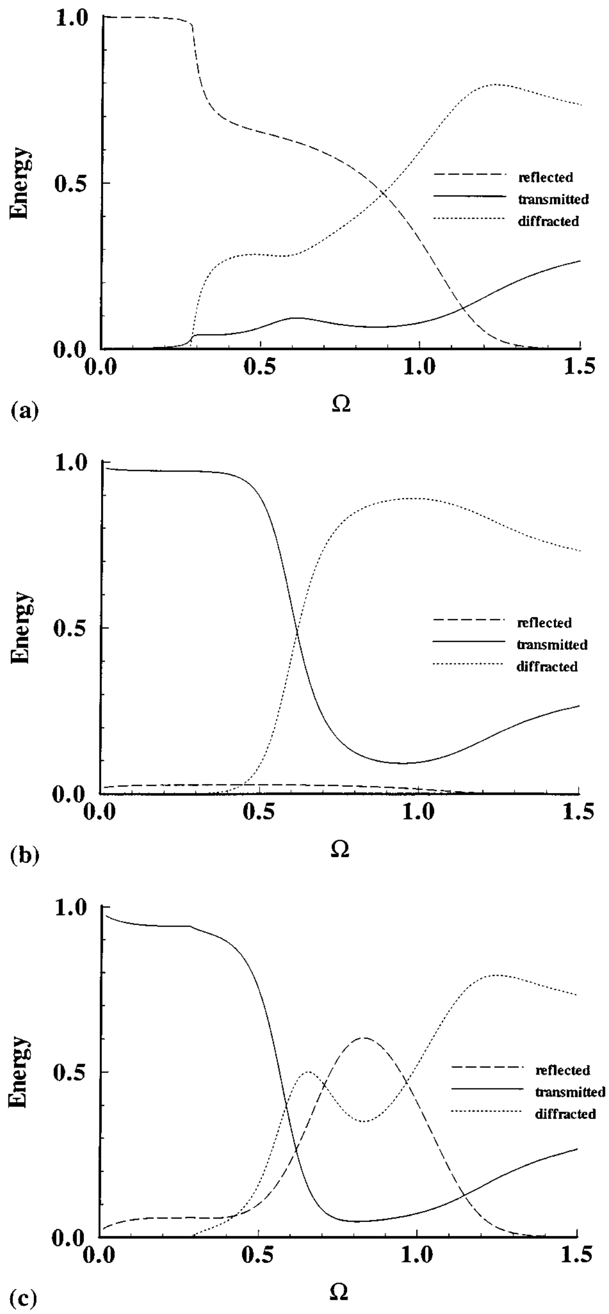

FIG. 5. The energy redistribution at a three member junction as frequency is varied for an oblique angle of incidence of 30°. (a) Infinite rib impedance, (b) zero rib impedance, and (c) finite rib impedance.

critical angle $\varphi_p^{\text{crit}}(\sim 45°)$. Finally, the components of energy are displayed as functions of frequency for an angle of incidence of 30° in Fig. 5.

V. CONCLUSION

We have derived explicit expressions for the interaction of an incident acoustic or plate wave with a three-member junction. The main results are in Eqs. (38) and (57), which determine the acoustic pressure in the surrounding fluid. The quadratic function $A(\xi)$ of Eq. (57) contains all the information about the obstruction and the fluid structure interaction at the junction. When the rib impedance is zero or infinite the general solution reduces to that of a pair of plates either welded or clamped together. Explicit formulae for the structural scattering coefficients have been computed for various parameter ranges. It is found that transmission of an incident subsonic flexural wave is highly dependent upon the angle of approach and the nature of the obstruction. In general, transmission is fully suppressed when the angle of incidence is greater than the critical angle φ_p^{crit} relating the two subsonic wave numbers of the plates. As the rib impedance is varied, transmission is greater for no rib in comparison to a rigid rib, and the amount of transmitted energy for a rib of finite impedance is intermediate between these. When the rib impedance is finite, the stiffness of the reinforcement dictates the amount of transmission.

ACKNOWLEDGMENT

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APPENDIX: FACTORIZATION OF $K(\xi)$

A semianalytical form for K^+ Refs. 13, 14 is

$$K^+(\xi)=\frac{\Pi'(1+\xi/\xi_n^{(2)})}{\Pi'(1+\xi/\xi_n^{(1)})}\left[\frac{D_2(0)}{D_1(0)}\right]^{1/2} \times \exp\left[\phi_1(\xi)-\phi_2(\xi)\right], \tag{A1}$$

where the products Π' are taken only over the three roots for which $s_n=1$, and

$$\phi(\xi)=\frac{1}{2\pi}\int_{\pi/2}^{\cos^{-1}(\xi/\bar{k})}\sum_{n=1}^{5}\left(\frac{\theta\sin\theta_n-\theta_n\sin\theta}{\cos\theta-\cos\theta_n}+\frac{\theta\sin\theta_n-(\pi-\theta_n)\sin\theta}{\cos\theta+\cos\theta_n}\right)s_n\,d\theta, \tag{A2}$$

with $s_n=1-D_j(\xi_n)$ and $\theta_n=\cos^{-1}(\xi_n/\bar{k})$. Here, $\pm\xi_n$, $n=1,2,...,5$, are the zeroes of $P(\xi)$ such that ξ_n are in $\mathscr{H}^+$, with no loss in generality, and P is the rationalized form of the dispersion relation for either plate, given by Eq. (66). The branch of the inverse cosine is $\cos^{-1}(\xi/\bar{k}) = i\log[\xi/\bar{k}+\gamma(\xi)/\bar{k}]$, where the principal branch of the logarithm is taken, $-\pi<\operatorname{Im}\log(\cdot)<\pi$. The form in (A1) is used for practical calculations because it does not have any possibly ambiguous square root functions in the preexponent, and the integrand is smooth.

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