

ACOUSTIC RADIATION GENERATED BY LOCAL EXCITATION OF SUBMERGED BEAMS AND STRINGS

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Free and forced motions of submerged one-dimensional waveguides have been investigated. Both the submerged beam and the submerged string can support a wave system which includes a pure surface wave in the fluid adjoining the waveguide, provided that certain conditions on the frequency (for the beam) or the physical parameters of the system (for the string) are satisfied. Dispersion curves are presented for steel beams and steel strings in water. Steady-state solutions have been derived for excitation by a concentrated time-harmonic load. The displacement responses at the point of application of the load and in the far field have been examined. Polar plots display the radiated far field in the fluid. At low frequencies the polar plots are circular in planes containing the waveguide, but the radiation pattern develops lobes as the frequency increases. Reflection, transmission and scattering into the fluid of an incident wave system by an elastic support of stiffness κ and mass m have also been examined, and the possibility of resonance phenomena has been investigated.

1. INTRODUCTION

The analysis of the structural response of a submerged solid body to local excitation by a concentrated force, together with an investigation of the associated acoustic radiation into the fluid, is central to studies of structure-fluid interaction. For the simplest geometry, namely, a fluid-loaded infinite elastic plate, the elastic response and the radiated acoustic field due to line-load or point-load excitation, has been discussed by several authors (see references [1-11]). It appears, however, that the corresponding problem for a slender body, such as a string or a beam, which is totally submerged in a fluid has not been treated in as much detail. Free waves in a submerged string have been discussed in reference [2], while approximate results for a beam have been presented in references [12] and [13].

In the present paper we augment the literature on wave motion in submerged strings and beams with results for (1) free waves, (2) response to local excitation, (3) acoustic radiation due to local excitation, and (4) scattering by an elastic support with mass. Where significant differences exist we contrast the results for the string with those for the beam. Both a solid beam and a hollow one (a pipe) are considered.

For free time-harmonic waves the submerged beam can support a wave system consisting of a flexural wave in the beam and a pure surface wave in the fluid adjoining the beam, provided that the frequency is smaller than a cut-off frequency. Above the cut-off frequency the surface wave becomes leaky, since part of its energy propagates radially from the beam. A similar wave system can be supported by the string, but only if a certain condition on the physical parameters of the string and the fluid is satisfied.

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When the condition for the existence of pure surface waves is satisfied, these waves can be generated by concentrated force excitation. At very low frequencies virtually all the input energy is, however, converted into wave motion of the string or the beams. Still at low frequencies some of the input energy is carried away by surface waves. A substantial percentage of the input energy is, however, converted into spherically radiating wave motion as the cut-off frequency is approached. Polar plots are presented for the radiation function. At low frequencies the polar plots are circular, but the radiation pattern develops lobes as the frequency increases.

The results for the concentrated-load response can be used to investigate the reflection, transmission and scattering of an incident wave system by an elastic support of stiffness κ and mass m . It is shown that an incident wave of frequency $\omega = (\kappa/m)^{1/2}$ passes unhindered by the support point of the mass-spring systems, since the interaction force vanishes at the natural frequency of the support. There is, however, a (bounded) resonance effect at frequencies slightly less than $(\kappa/m)^{1/2}$.

2. EQUATIONS OF MOTION

The transverse motion of a submerged string is governed by (see Figure 1)

$$\rho A \partial^2 w / \partial t^2 - T \partial^2 w / \partial x^2 = P(x, t), \quad (2.1)$$

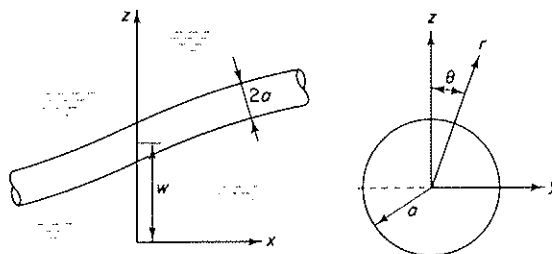


Figure 1. Submerged string or beam of circular cross-section.

where T is the tension force. The corresponding equation for a submerged Bernoulli-Euler beam is

$$\rho A \partial^2 w / \partial t^2 + EI \partial^4 w / \partial x^4 = P(x, t), \quad (2.2)$$

where EI is the bending stiffness. In both equations ρ denotes the mass density/unit volume and A is the cross-sectional area, while $P(x, t)$ denotes the transverse force per unit length. For a string or a beam of circular cross-section, the fluid pressure $p(r, \theta, x, t)$, $r \geq a$, gives rise to a transverse force of the form

$$P_f(x, t) = - \int_0^{2\pi} p(a, \theta, x, t) \cos \theta a d\theta. \quad (2.3)$$

Acoustic waves in an ideal fluid are governed by

$$\nabla^2 \phi = (1/c_f^2) \ddot{\phi}, \quad \dot{\mathbf{u}} = \nabla \phi \quad \text{and} \quad p = -\rho_f \dot{\phi}, \quad (2.4a-c)$$

where c_f is the sound speed, $\mathbf{u}$ is the particle velocity and p denotes the pressure, while ρ_f is the mass density of the fluid.

Interaction with the fluid takes place via continuity of the *radial* component of the particle velocity and via the fluid pressure. The action of the fluid pressure is represented

by equation (2.3). Continuity of the radial particle velocity yields

$$\dot{w}(x, t) \cos \theta = (\partial \phi / \partial r)|_{r=a}. \quad (2.5)$$

We will also consider the case that the beam is actually a thin-walled pipe, whose interior is filled with fluid. Equation (2.3) must then be replaced by

$$P_f(x, t) = \int_0^{2\pi} [p_i(a, \theta, x, t) - p(a, \theta, x, t)] \cos \theta a d\theta, \quad (2.6)$$

where $p_i(r, \theta, x, t)$, $r \leq a$, is the fluid pressure inside the pipe.

The response of the string or the beam to a time-harmonic concentrated load of magnitude F_0 , applied at $x=0$, will be considered in some detail. The transverse force then is of the form

$$P(x, t) = P_f(x, t) + F_0 \delta(x) \exp(-i\omega t), \quad (2.7)$$

where $\delta(x)$ is the Dirac delta function.

3. FREE WAVES AND DISPERSION EQUATIONS

The propagation of free waves along a string immersed in a fluid has been investigated in reference [2]. The analysis of free transverse waves propagating along a submerged Bernoulli-Euler beam of circular cross-section proceeds along very similar lines. In this section we briefly consider both cases, and indicate some differences in the dispersive behaviors.

A time-harmonic transverse wave propagating along a string or a beam may be represented by

$$w(x, t) = C \exp[i(kx - \omega t)], \quad (3.1)$$

where C is a constant. Equations (2.5) and (3.1) suggest a corresponding expression for the acoustic potential of the form

$$\phi(r, \theta, x, t) = \Phi(r) \cos \theta \exp[i(kx - \omega t)]. \quad (3.2)$$

Substitution of equation (3.2) into equation (2.4a) yields a Bessel equation for $\Phi(r)$. The appropriate solution, which satisfies both the radiation condition and the interface condition (2.5), is of the form

$$\Phi(r) = -i\omega C H_1^{(1)}(qr) / q H_1^{(1)'}(qa), \quad (3.3)$$

where $H_1^{(1)}(qr)$ is the Hankel function of the first kind of order one. Also

$$q^2 = k_f^2 - k^2, \quad k_f^2 = \omega^2 / c_f^2. \quad (3.4a, b)$$

Substitution of equation (3.2) into equation (2.4c), and then in equation (2.3), and subsequent substitution of the result together with equation (3.1) into equation (2.1), yields a relation between ω and k . In dimensionless form this dispersion equation may be written as

$$D_s(\xi, \Omega) = 0, \quad \text{where} \quad D_s(\xi, \Omega) = \xi^2 - \Omega^2 + \epsilon \Omega^2 H_1^{(1)}(\bar{q}) / \bar{q} H_1^{(1)'}(\bar{q}). \quad (3.5, 3.6)$$

In equation (3.6),

$$\Omega^2 = (\omega a / c_s)^2, \quad c_s^2 = T / \rho A, \quad \xi = ka, \quad (3.7a-c)$$

$$\alpha^2 = c_s^2 / c_f^2, \quad \epsilon = \rho_f / \rho \quad (3.8a, b)$$

$$\bar{q} = qa = (\alpha^2 \Omega^2 - \xi^2)^{1/2}. \quad (3.9)$$

Here c_s is the phase velocity of waves in the free string.

For given frequency (wavenumber), equation (3.5) is a transcendental equation for the wavenumber (frequency). It is of particular interest to consider the case that ξ is real-valued, and

$$\alpha^2 \Omega^2 - \xi^2 < 0. \quad (3.10)$$

In that case the Hankel function $H^{(1)}(\bar{q})$ converts to a modified Bessel function, and instead of equation (3.3) one has

$$\Phi(r) = -i\omega Ca K_1[|\bar{q}|(r/a)]/|\bar{q}|K'_1(|\bar{q}|), \quad (3.11)$$

where $K_1(\)$ denotes the modified Bessel function. The expression (3.11) for $\Phi(r)$, in conjunction with expression (3.2), then corresponds to a wave motion of the fluid which propagates in the x -direction, and whose amplitude decays in the r -direction. Energy propagation occurs in the x -direction only. This wave motion is of the nature of a surface wave. When condition (3.10) is satisfied, the terms in $D(\xi, \Omega)$ become real valued, and equation (3.5) takes the form

$$\xi^2 - \Omega^2 + \epsilon \Omega^2 K_1(|\bar{q}|)/|\bar{q}|K'_1(|\bar{q}|) = 0. \quad (3.12)$$

It can be shown that real-valued solutions of equation (3.12) exist for

$$\alpha^2 \leq \alpha_c^2, \quad \text{where} \quad \alpha_c^2 = 1 + \epsilon. \quad (3.13a, b)$$

It may be verified that

$$\xi = \alpha_c \Omega \quad \text{for} \quad \alpha^2 = \alpha_c^2. \quad (3.14)$$

No solutions with the property (3.10) exist when $\alpha^2 > \alpha_c^2$. It is noted that the magnitude of α is primarily controlled by the tension force in the string. A critical value T_c follows from equations (3.7b), (3.8a) and (3.13b) at $T_c = (1 + \epsilon)c_f^2 \rho A$. When $T < T_c$ there is a plane wave of the type (3.11) propagating in the x -direction in the fluid. For $T = T_c$ one finds $\Phi(r) \propto (a/r)$. When $T > T_c$, $\Phi(r)$ is of the form (3.3), and energy propagates in the r -direction as well as in the x -direction.

For the Bernoulli-Euler beam an equation of the form (3.5) can be derived in a completely analogous manner. One finds

$$D_b(\xi, \Omega) = 0, \quad \text{where} \quad D_b(\xi, \Omega) = \xi^4 - \Omega^2 + \epsilon \beta \Omega^2 H_1^{(1)}(\bar{q})/\bar{q} H_1^{(1)'}(\bar{q}). \quad (3.15, 3.16)$$

In equation (3.16)

$$\Omega^2 = (\omega a / c_b)^2, \quad c_b^2 = EI / \rho A a^2, \quad (3.17a, b)$$

and

$$\alpha^2 = c_b^2 / c_f^2, \quad \beta = \pi a^2 / A. \quad (3.18a, b)$$

The constants ξ , ϵ and $\bar{q}$ are defined by equations (3.7c), (3.8b) and (3.9), respectively. The constant β ($\beta \geq 1$) is introduced to account for the possibility that the beam may be hollow (a pipe).

When $\alpha^2 \Omega^2 - \xi^2 < 0$, the wave motion in the fluid is a surface wave, represented by expression (3.11). The dispersion equation (3.15) then becomes

$$\xi^4 - \Omega^2 + \epsilon \beta \Omega^2 K_1(|\bar{q}|)/|\bar{q}|K'_1(|\bar{q}|) = 0. \quad (3.19)$$

For $\xi^2 = \alpha^2 \Omega^2$, equation (3.19) reduces to the form

$$\alpha^4 \Omega^4 - \Omega^2 - \varepsilon \beta \Omega^2 = 0, \quad (3.20)$$

with solutions

$$\Omega = \Omega_c = (1 + \varepsilon \beta)^{1/2} / \alpha^2, \quad \xi = \xi_c = (1 + \varepsilon \beta)^{1/2} / \alpha. \quad (3.21a, b)$$

For $\Omega \leq \Omega_c$, equation (3.19) has a pair of real roots, but for $\Omega > \Omega_c$, equation (3.19) has no real roots. The solution for $\Phi(r)$ is then of the form (3.3). The cut-off frequency Ω_c depends on ε , β and α . In the limit case of vanishing fluid, $\varepsilon \rightarrow 0$, $\alpha \rightarrow 0$, one has $\Omega_c \rightarrow \infty$, $\xi_c \rightarrow \infty$.

The results that have been obtained in this section point to an interesting difference between the dispersive behaviors of the submerged string and the submerged Bernoulli-Euler beam. For both cases a necessary condition for pure surface waves is that $\alpha^2 \Omega^2 \leq \xi^2$. For the beam the value of α is arbitrary, but for given ε , β and α there is critical frequency given by equation (3.21a), which provides the upper limit of frequencies for surface waves in the fluid. For the string, in addition to $\alpha^2 \Omega^2 \leq \xi^2$ one must require $\alpha^2 \leq \alpha_c^2$, which corresponds to a critical value of certain physical parameters (for example, the tension force). However, when $\alpha^2 \leq \alpha_c^2$ there is always a surface wave in the domain $\alpha^2 \Omega^2 \leq \xi^2$; i.e., there is no cut-off frequency.

For $\beta > 1$, the beam of circular cross section corresponds to a pipe. If the fluid also fills the interior of the pipe, equation (2.7) should be used to represent the transverse force on the beam. In addition to equation (3.3), or equation (3.11), one now has, for $r < a$, and real-valued ξ , Ω ,

$$\Phi(r) = \begin{cases} -i\omega C J_1(qr) / q J_1'(\bar{q}), & \xi^2 < \alpha^2 \Omega^2 \\ -i\omega C I_1(|q|r) / |q| I_1'(|\bar{q}|), & \xi^2 > \alpha^2 \Omega^2 \end{cases} \quad (3.22a, b)$$

where $J_1(\)$ and $I_1(\)$ are the ordinary Bessel function and the modified Bessel function of the first kinds. For real-valued ξ , Ω , and for $\xi^2 \geq \alpha^2 \Omega^2$, substitution of equations (3.22b) and (3.11) into equations (3.2) and (2.4c), and subsequent substitution of the result into equation (2.6), yields P_f . The corresponding dispersion function then follows by substitution of P_f and equation (3.1) into equation (2.2). By using the relation $K_1(z)I_1'(z) - I_1(z)K_1'(z) = 1/z$, one finds

$$D_p(\xi, \Omega) = \xi^4 - \Omega^2 + \varepsilon \beta \Omega^2 / |\bar{q}|^2 I_1'(|\bar{q}|) K_1'(|\bar{q}|). \quad (3.23)$$

For all other cases one has

$$D_p(\xi, \Omega) = \xi^4 - \Omega^2 - 2i\varepsilon \beta \Omega^2 / \pi \bar{q}^2 J_1'(\bar{q}) H_1^{(1)'}(\bar{q}). \quad (3.24)$$

Numerical results have been obtained for a steel string and a steel beam in water. The ratio of the mass densities of water and steel is $\varepsilon = 0.128$. For the string one finds from equation (3.13b) that $\alpha_c^2 = 1.128$. For a steel beam of solid circular cross section $\beta = 1$, and $I/A = a^2/4$. Equation (3.18a) then yields $\alpha^2 = 2.957$. It then follows from equations (3.21a, b) that $\Omega_c = 0.359$, and $\xi_c = 0.618$. For a pipe one has $I/A = \frac{1}{2}a^2$, and $\alpha^2 = 5.913$, and we have chosen $\beta = 2\pi$ (which corresponds to a wall thickness $h \sim 0.08a$). Then $\Omega_c = 0.227$ and $\xi_c = 0.552$. When the interior of the pipe is also filled with water, the latter geometry gives $\Omega_c = 0.273$ and $\xi_c = 0.664$. Figure 2a shows dispersion curves for the submerged string for the case of surface waves in the water ($\alpha \leq \alpha_c$). Dispersion curves for the beam in the frequency range of surface waves in the water are shown in Figure 2b. These figures also show the relation $\xi_f = \alpha \Omega$ for comparison. For the beam the intersection point of the curve of ξ versus Ω and the line $\xi_f = \alpha \Omega$ defines the cut-off frequency for pure surface waves in the fluid.

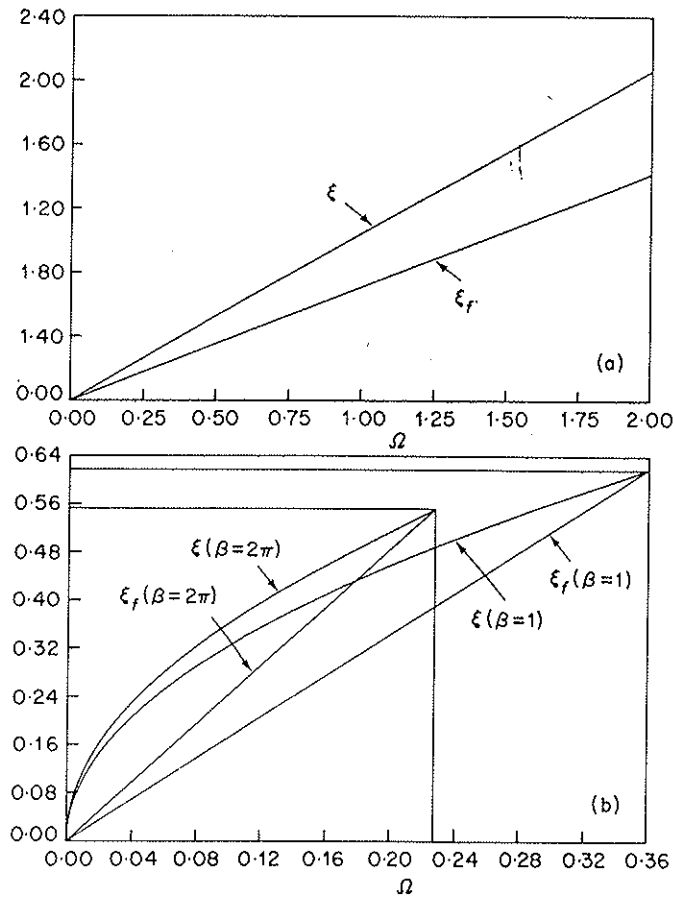


Figure 2. Curves relating dimensionless wavenumber, $\xi = ka$, to dimensionless frequency, $\Omega = \omega a / c_s$ or $\Omega = \omega a / c_b$. (a) String: $\alpha^2 = 0.5$; (b) beam: $\beta = 1$ (solid beam), $\alpha^2 = 2.957$ and $\beta = 2\pi$ (pipe), $\alpha^2 = 5.913$.

4. FORCED MOTION

A steady-state solution for loading by a concentrated time-harmonic force can easily be obtained by the use of Fourier integral methods. Since the Dirac delta function has the integral representation

$$\delta(x) = \frac{1}{a} \delta(x/a) = \frac{1}{2\pi a} \int_{-\infty}^{\infty} e^{i(x/a)\xi} d\xi, \quad (4.1)$$

the steady-state solution to equation (2.1), or equation (2.2), with $P(x, t)$ defined by equation (2.7), can be written as

$$w(x, t) = W(x) e^{-i\omega t}, \quad \text{where} \quad W(x) = \frac{a}{2\pi} \bar{F}_0 \int_{-\infty}^{\infty} \frac{e^{i(x/a)\xi}}{D(\xi, \Omega)} d\xi. \quad (4.2, 4.3)$$

In equation (4.3)

$$\bar{F}_0 = F_0/T, \quad D(\xi, \Omega) = D_s(\xi, \Omega) \quad \text{for the string, see equation (3.6),} \quad (4.4)$$

$$\bar{F}_0 = F_0 a^2/EI, \quad D(\xi, \Omega) = D_b(\xi, \Omega) \quad \text{for the beam, see equation (3.16).} \quad (4.5)$$

The corresponding expression for the acoustic potential is

$$\phi(r, \theta, x, t) = \Phi(r, x) \cos \theta e^{-i\omega t}, \quad (4.6)$$

where

$$\Phi(r, x) = -\frac{i\omega a^2}{2\pi} \bar{F}_0 \int_{-\infty}^{\infty} \frac{H_1^{(1)}[(r/a)\bar{q}] e^{i(x/a)\xi}}{\bar{q} H_1^{(1)}(\bar{q}) D(\xi, \Omega)} d\xi. \quad (4.7)$$

In both equations (4.3) and (4.7), the branch cuts are chosen such that $\text{Im}[(\alpha^2 \Omega^2 - \xi^2)^{1/2}] > 0$.

The displacement amplitude under the applied load follows from equation (4.3) as

$$W(0) = \bar{F}_0 a f(\Omega), \quad \text{where} \quad f(\Omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{d\xi}{D(\xi, \Omega)}. \quad (4.8, 4.9)$$

The integral in equation (4.9) is complex valued. It can be reduced to a sum of real-valued integrals by using contour deformation and the residue theorem. In the present paper we have, however, opted for a direct numerical integration of equation (4.9). For both the string and the beam, the analysis of the dispersion equations has revealed that for $\alpha^2 \leq \alpha_c^2$ (string) and $\Omega \leq \Omega_c$ (beam) two real zeros of $D(\xi, \Omega)$ exist on the real axis, say at $\xi = \pm \xi_0$. Hence one has to compute the Cauchy principal value of the integral, and add πi times the residue at $\xi = \xi_0$ and $-\pi i$ times the residue at $\xi = -\xi_0$. It is easy to show that $D(\xi, \Omega)$ is an even function of ξ , while $dD(\xi, \Omega)/d\xi$ is an odd function. As before, equation (4.9) is valid for the string and the beam by using $D_s(\xi, \Omega)$ and $D_b(\xi, \Omega)$, respectively. Numerical results for the dimensionless displacement under the load, namely, the modulus of $f(\Omega)$, $|f(\Omega)| = (1/\bar{F}_0 a)|W(0)|$, and its phase, are shown in Figures 3 and 4. For both the submerged string and the submerged beam, the magnitude of the displacement under the

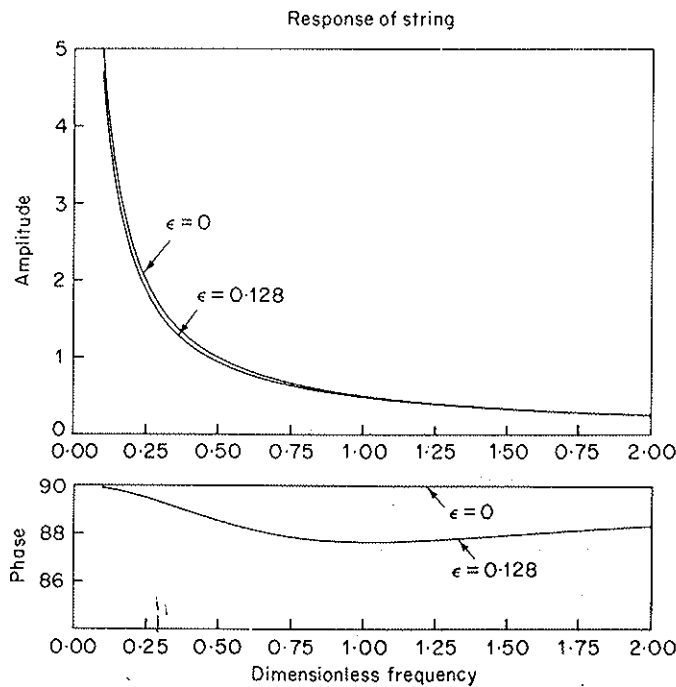


Figure 3. Displacement response of the string at the point of application of the concentrated load, $\alpha^2 = 0.8$.

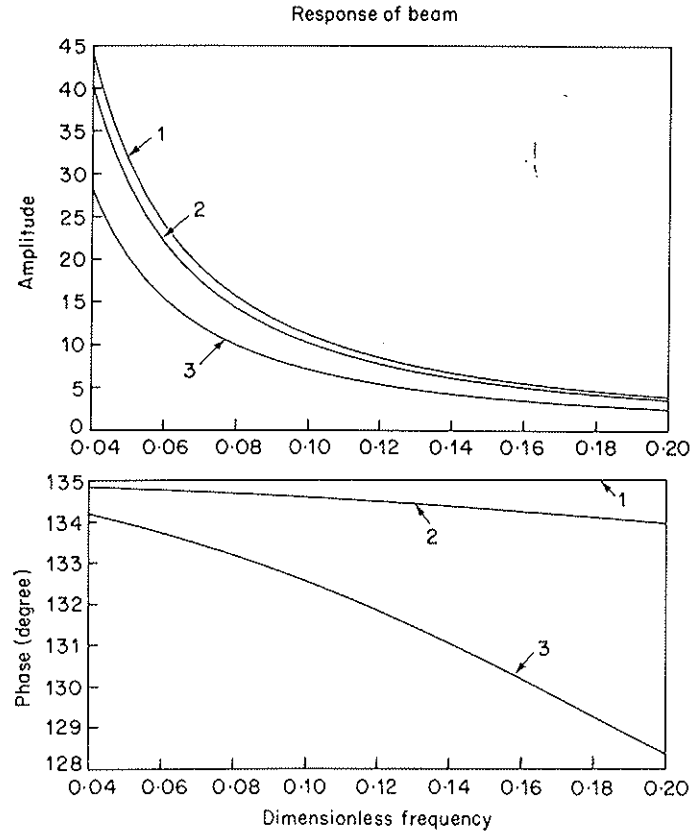


Figure 4. Displacement response of the beam and the pipe at the point of application of the concentrated load. 1, $\varepsilon = 0$; 2, $\varepsilon = 0.128$, $\beta = 1$, $\alpha^2 = 2.957$ (solid beam); 3, $\varepsilon = 0.128$, $\beta = 2\pi$, $\alpha^2 = 5.913$ (pipe).

load decreases with increasing frequency. The curve for $\varepsilon = 0$ represents the response in vacuum. The presence of water ($\varepsilon = 0.128$) reduces the displacement magnitude.

For sufficiently large values of $|x|/a$, the principal contributions to the integral in equation (4.3) come from the poles on the real axis. One finds

$$W(x) \sim i\bar{F}_0 a \left[\frac{d}{d\xi} D(\xi, \Omega) \right]_{\xi=\xi_0}^{-1} e^{i\xi_0(|x|/a)}. \quad (4.10)$$

As before, this result is valid for the string as well as for the beam by appropriate substitution of $\bar{F}_0$ and $D(\xi, \Omega)$ from either equation (4.4) or equation (4.5). Only for the submerged beam is the far field displacement noticeably different from the one under the load. Numerical results for $(1/\bar{F}_0 a)|W(x)|$ are shown in Figure 5. In the regions of interest, defined by $\alpha^2 \leq \alpha_c^2$ (string) and $\Omega \leq \Omega_c$ (beam), the surface-wave motion in the fluid which accompanies the string or beam wave (4.10) follows from equations (3.2) and (3.11) by substituting $\xi = \xi_0$.

It remains to consider the spherical wave motion in the fluid, which radiates from the point of application of the concentrated force. A far field expression can be obtained from equation (4.7). One starts by introducing the asymptotic approximation to the Hankel function for large values of r/a ,

$$H_1^{(1)}[(r/a)\bar{q}] \cong (2a/\pi\bar{q}r)^{1/2} e^{i[(r/a)\bar{q} - 3\pi/4]} \quad (4.11)$$

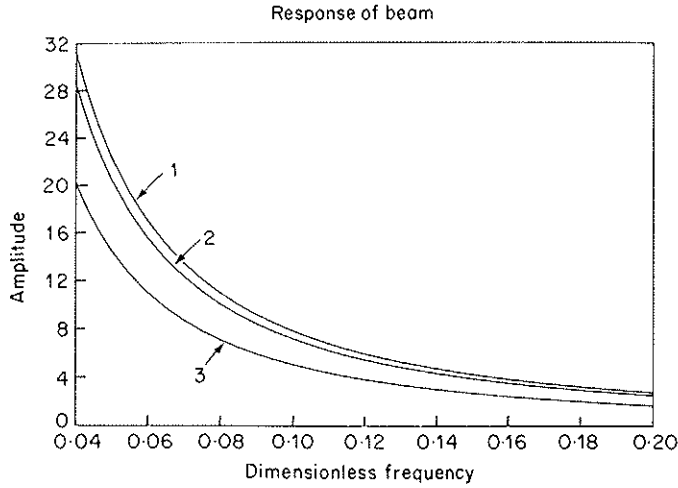


Figure 5. Far field deflection for concentrated load excitation of the beam. 1, Beam in vacuum; 2, solid beam in water; 3, pipe in water.

in equation (4.7). Next one introduces the polar angle ψ and the spherical radius R by the relations

$$r = R \sin \psi, \quad x = R \cos \psi. \quad (4.12a, b)$$

A subsequent change of variables in equation (4.7), defined by

$$\xi = \alpha \Omega \cos \chi, \quad \bar{q} = \alpha \Omega \sin \chi, \quad (4.13a, b)$$

puts the integral in a form which for large values of R/a is suitable for approximate evaluation by the method of steepest descent. The result may be written in the form

$$\Phi(R, \psi) \sim (i/\pi) \omega a^2 \bar{F}_0 \mathcal{R}(\psi) (e^{ik_f R}/k_f R), \quad (4.14)$$

where $\mathcal{R}(\psi)$ is the radiation function

$$\mathcal{R}(\psi) = [\sin \psi H_1^{(1)'}(\alpha \Omega \sin \psi) D(\xi, \Omega)|_{\xi = \alpha \Omega \cos \psi}]^{-1}. \quad (4.15)$$

Again, the results of equations (4.14) and (4.15) are valid for the string and the beam by using $\bar{F}_0$ and $D(\xi, \Omega)$ as defined by equations (4.4) and (4.5), respectively.

Equation (4.15) may be compared with the corresponding radiation function for a concentrated force in an unbounded fluid (see reference [14], p. 167):

$$\Phi(R, \psi) = (F_0 \omega / 4 \pi \rho_f c_f^2) \sin \psi (e^{ik_f R}/k_f R). \quad (4.16)$$

A polar plot of $(\text{constant}) \times \sin \psi$ is a perfect circle. For the pipe, polar plots of $|\mathcal{R}(\psi)|$ are shown in Figure 6 for several values of Ω . The corresponding plots for the string are shown in Figure 7. At low frequencies the polar plots for the pipe are almost circular. As Ω increases, the radiation patterns develop lobes. Some interesting differences between the patterns for the pipe and the string are noted, particularly at higher frequencies.

A valuable check on the computations is obtained by the balance of time-averaged energy fluxes. A time-averaged energy flux is of the general form of one-half times the product of a "force" term and the complex conjugate of a "velocity" term, integrated over an appropriate surface. For the fluid, the force and velocity components are the pressure and the particle velocity component normal to the surface. For the beam they are the bending moment and the rate of change of the slope, and for the string they are

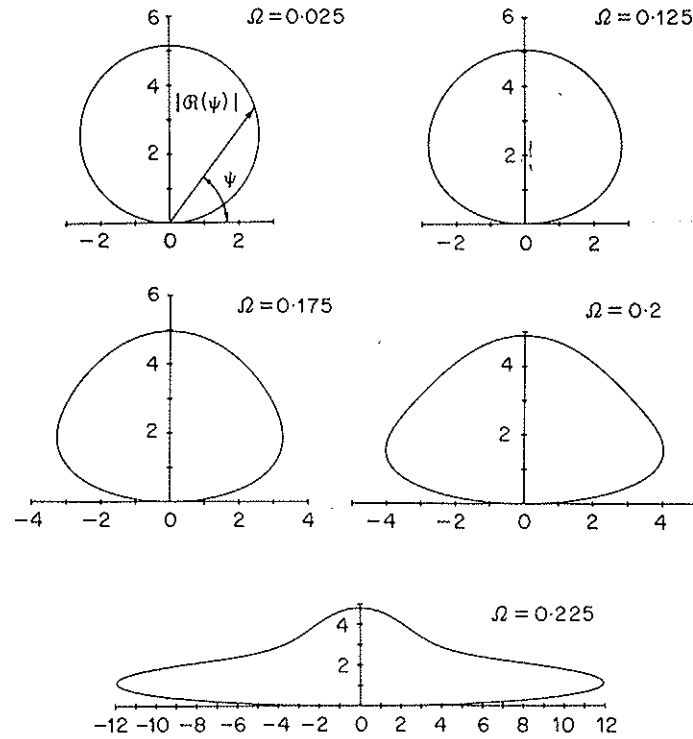


Figure 6. Radiation pattern in water for point-force excitation of a submerged steel pipe ($\beta = 2\pi$) for $\Omega = 0.025, 0.125, 0.175, 0.2$, and $\Omega = 0.225$.

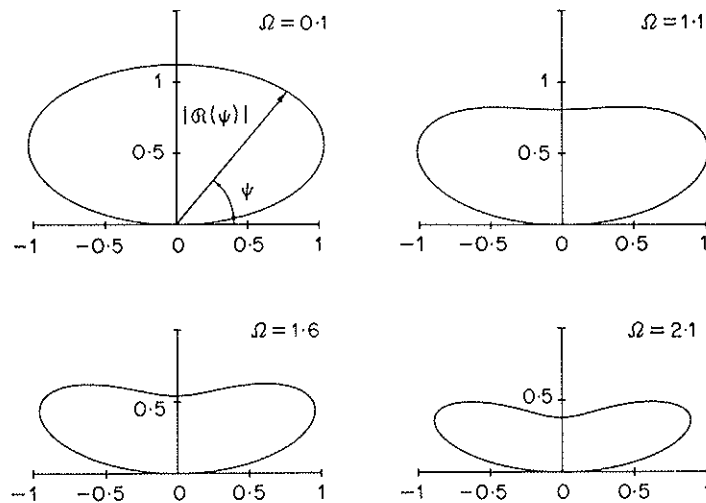


Figure 7. Radiation pattern in water for point-force excitation of a submerged steel string, $\alpha^2 = 0.8$, for $\Omega = 0.1, 1.1, 1.6$ and $\Omega = 2.1$.

the vertical component of the tensile force and the rate of change of the slope. The force applied at the origin on the string or the beam pumps energy into the system. The energy input averaged over a period of vibration is denoted by $\langle \mathcal{P}_i \rangle$. The time-averaged energy fluxes in the string and the beam are denoted by $\langle \mathcal{P}_s \rangle$ and $\langle \mathcal{P}_b \rangle$, respectively. In the fluid

we have time averaged energy flux due to surface waves $\langle \mathcal{P}_{sw} \rangle$, and due to radial radiation of body waves $\langle \mathcal{P}_R \rangle$. The balance of rates of energies for the string requires

$$\langle \mathcal{P}_i \rangle = \langle \mathcal{P}_s \rangle + \langle \mathcal{P}_{sw} \rangle + \langle \mathcal{P}_R \rangle. \quad (4.17)$$

An analogous equation holds for the beams. The pertinent expressions are

$$\langle \mathcal{P}_i \rangle = Q \left\{ \left[\frac{d}{d\xi} D(\xi, \Omega) \right]_{\xi=\xi_0}^{-1} - \frac{1}{\pi} \int_0^{\alpha\Omega} \frac{\text{Im}[D(\xi, \Omega)] d\xi}{\text{Re}^2[D(\xi, \Omega)] + \text{Im}^2[D(\xi, \Omega)]} \right\}, \quad (4.18)$$

$$\langle \mathcal{P}_s \rangle = 2Q\xi_0 \left[\frac{d}{d\xi} D(\xi, \Omega) \right]_{\xi=\xi_0}^{-2}, \quad \langle \mathcal{P}_b \rangle = 4Q\xi_0^3 \left[\frac{d}{d\xi} D(\xi, \Omega) \right]_{\xi=\xi_0}^{-2}, \quad (4.19, 4.20)$$

$$\langle \mathcal{P}_{sw} \rangle = 2Q\xi_0 \left[\frac{d}{d\xi} D(\xi, \Omega) \right]_{\xi=\xi_0}^{-2} \frac{\varepsilon\beta\Omega^2}{q_0^2 [K_1'(q_0)]^2} \int_1^\infty [K_1(q_0 v)]^2 v dv, \quad (4.21)$$

$$\langle \mathcal{P}_R \rangle = Q \frac{\varepsilon\beta\Omega}{\pi^2 \alpha} \int_0^\pi |\mathcal{R}(\psi)|^2 \sin \psi d\psi, \quad (4.22)$$

where

$$Q = \begin{cases} F_0^2 \omega a / 2T & \text{for the string} \\ F_0^2 \omega a^3 / 2EI & \text{for the beam} \end{cases}, \quad q_0 = (\xi_0^2 - \alpha^2 \Omega^2)^{1/2}. \quad (4.23a, b)$$

For the string $\beta = 1$, while for a pipe one has $\beta \gg 1$.

TABLE 1

Distribution of time-averaged energy input over flexural wave motion in the beam, $\langle \mathcal{P}_b \rangle$, surface wave motion in the water, $\langle \mathcal{P}_{sw} \rangle$, and spherically radiated wave motion, $\langle \mathcal{P}_R \rangle$, in percentage points

Ω	Solid beam: $\beta = 1$ $\Omega_c = 0.359$			Empty pipe: $\beta = 2\pi$ $\Omega_c = 0.227$		
	$\langle \mathcal{P}_b \rangle$	$\langle \mathcal{P}_{sw} \rangle$	$\langle \mathcal{P}_R \rangle$	$\langle \mathcal{P}_b \rangle$	$\langle \mathcal{P}_{sw} \rangle$	$\langle \mathcal{P}_R \rangle$
0.01	99.89	0.11	0.00	99.46	0.53	0.01
0.05	99.61	0.35	0.04	97.94	1.65	0.41
0.11	99.09	0.60	0.31	94.07	3.00	2.93
0.15				89.48	4.11	6.41
0.17	98.20	0.85	0.95	86.28	4.86	8.86
0.19				82.11	5.89	12.00
0.21				75.96	7.62	16.42
0.23	96.73	1.18	2.09			
0.26	95.69	1.41	2.90			
0.29	94.35	1.74	3.91			
0.32	92.46	2.27	5.27			
0.35	88.65	3.65	7.69			

Table 1 shows the distribution of the time-averaged energy input of the force, $\langle \mathcal{P}_i \rangle$, over flexural wave motion in the beam, surface wave motion in the water, and spherically radiated wave motion in the water. At very low frequencies, more than 99% of the input energy is converted into beam waves. This proportion decreases, however, as the frequency increases. Still at low frequencies, some energy is carried away by surface waves. At

frequencies quite close to the cut-off frequency, a substantial percentage of the input energy is converted into spherically radiating wave motion. The conversion of input energy into wave motion in the fluid is more pronounced for an empty pipe than for a solid beam.

5. REFLECTION AND TRANSMISSION AT SUPPORTS

The results of the preceding sections can be used to investigate the effects of an elastic support (with mass) on a time-harmonic wave which is propagating along the submerged string or the submerged beam. In the frequency range that is being considered, an incident wave involves surface-wave motion in the fluid. The wave motion will be reflected by the support, and transmitted beyond it. In addition spherically radiating scattered waves in the fluid will be generated at the support point. As a model of the elastic support with mass m , we consider a concentrated mass, which is fixed to the string or the beam, and which is also supported by a spring of constant κ (see Figure 8).

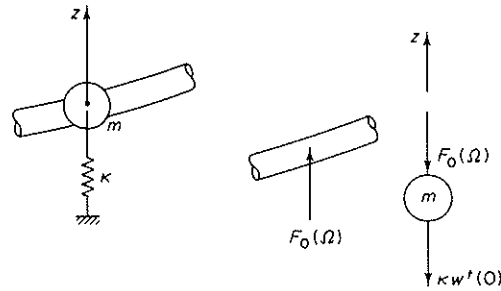


Figure 8. Spring-mass support point.

The system of incident time-harmonic waves is defined by equation (3.1), and equations (3.2) and (3.11). The total fields are expressed as sums of incident and scattered fields,

$$W'(x) = W^i(x) + W^s(x), \quad \Phi'(r, x) = \Phi^i(r, x) + \Phi^s(r, x), \quad (5.1, 5.2)$$

where $\exp(-i\omega t)$ has been omitted from both equations (5.1) and (5.2) and the term $\cos \theta$ has been left out in equation (5.2). In the present idealization the effect of the mass-spring system is equivalent to a point load on the string. Hence $W^s(x)$ and $\Phi^s(r, x)$ are of the general forms given by equations (4.3) and (4.7). For the present problem F_0 depends, however, on the frequency, and $F_0(\Omega)$ must be determined from the interaction between the string (or the beam) and the mass-spring system.

The equation of motion of the mass m may be written as

$$-m\omega^2 W'(0) = -\kappa W'(0) - F_0(\Omega), \quad (5.3)$$

where we have used the fact that the displacement of the mass equals the displacement of the string or the beam at the support point. By combining equations (5.1), (3.1) and (4.8), one finds

$$W'(0) = C + F_0(\Omega) \gamma f(\Omega), \quad (5.4)$$

where C is the amplitude of the incident wave, and

$$\gamma = a/T \quad \text{for the string,} \quad \gamma = a^3/EI \quad \text{for the beam.} \quad (5.5, 5.6)$$

Substitution of equation (5.4) into equation (5.3) yields

$$F_0(\Omega) = -(1 - \omega^2/\omega_*^2)\kappa C/[1 + \kappa\gamma(1 - \omega^2/\omega_*^2)f(\Omega)]. \quad (5.7)$$

Here ω_* is the natural frequency of the supporting mass-spring system

$$\omega_*^2 = \kappa/m. \quad (5.8)$$

Far away the scattered wave $W^s(x)$ is given by equation (4.10), and equations (4.4) and (4.5), with F_0 defined by equation (5.7). The spherical radiation in the fluid is given by equations (4.14), (4.15), (4.4) and (4.5) and (5.7). The amplitude and the phase of the interaction force $F_0(\Omega)$ have been plotted in Figure 9. It is of interest to note that $F_0(\Omega)$

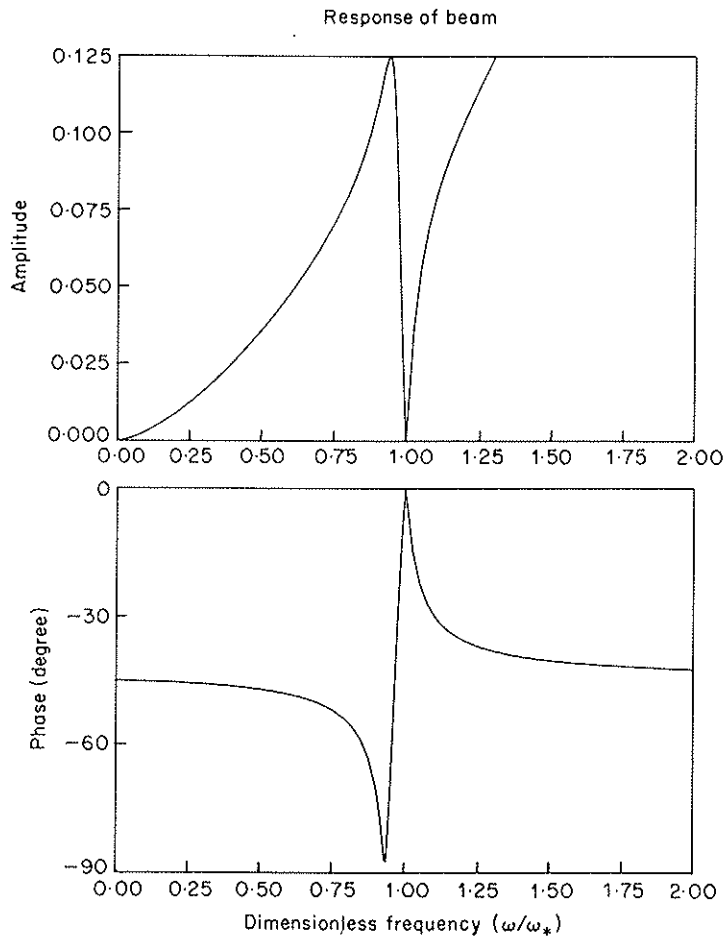


Figure 9. Amplitude and phase of the interaction force $F_0(\omega/\omega_*)$ for a solid steel beam and mass-spring support in water; $\kappa\gamma=1$, $\rho Aa/m=0.01$.

vanishes when the frequency of the incident wave equals the natural frequency of the mass-spring system. Hence an incident wave of frequency $\omega = \omega_*$ passes unhindered by the support point of the mass-spring system.

The total displacement at the support point follows from equations (5.1), (4.8) and (5.7) as

$$W'(0)/W'(0) = 1/[1 + \kappa\gamma(1 - \omega^2/\omega_*^2)f(\Omega)] \quad (5.9)$$

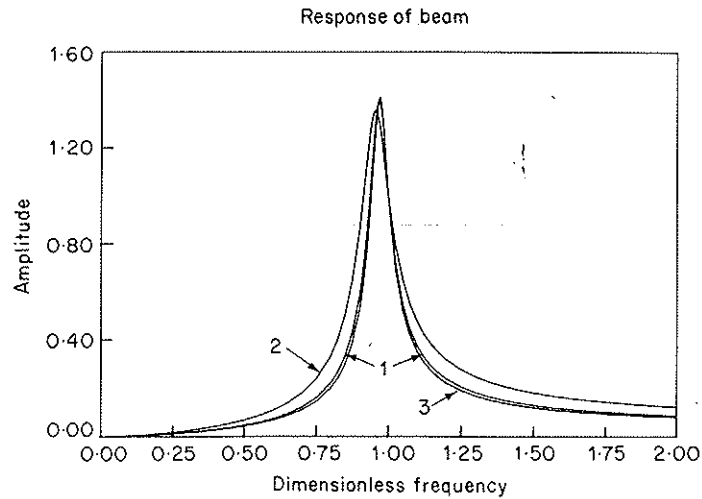


Figure 10. Response at support point, $|W'(0)/W^i(0)|$, for a wave incident on a mass-spring support in water; $\kappa\gamma = 1$, $\rho Aa/m = 0.01$. 1, $\varepsilon = 0$; 2, $\varepsilon = 0.128$, $\beta = 2\pi$; 3, $\varepsilon = 0.128$, $\beta = 1$.

where γ is defined by either equation (5.5) or equation (5.6). For the beam $|W'(0)/W^i(0)|$ has been plotted versus ω/ω_* in Figure 10. It is noted that there is a resonance effect for frequencies slightly less than ω_* . The shift of the resonance frequency is due to the effect of the fluid on the system.

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