

Spontaneous tribocharging of similar materials

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Abstract – We investigate the spontaneous triboelectrification of similar materials. This effect, first reported in 1927, has been little studied but is easily reproduced. We find in two separate experimental systems, where materials are prepared in the same way and rubbed symmetrically, that symmetry breaking occurs so that one material becomes positive and the other negative. Curiously, the distribution of charges on the materials appears to be self-similar, with different charge patterns on the positive and the negative surface. We propose a mechanism in which an initial localized charge may spawn the production of smaller localized charges of the same polarity.

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Introduction. – It has been recognized since the 16th century [1,2] that insulators (originally termed “electrics”) can readily be charged by rubbing or contact, while conductors (“non-electrics”) are more difficult to charge. Careful experiments have shown this effect to be distinct from the ability of conductors to carry acquired charges to ground [3]. Thus PTFE—one of the best insulators known—is the material of choice to produce mega-Volt potentials in Van de Graaf generators. This is paradoxical since insulators lack free charge carriers, and it has remained unexplained to this day how they recruit these charges. The paradox has been compounded by experiments first performed in the 1920s [4] and later rigorously repeated [5,6] that demonstrate that even identical insulators can transfer charges during rubbing.

In one sense, this should come as little surprise, since charge is also generated through contact between identical materials in the best known electrification system: atmospheric lightning. Similarly, large charges are acquired by aeolian transport of desert sands, which have little to rub against beyond other sand [7–9]. It is troublesomely unclear how insulators in general acquire the free charges that they convey during tribocharging, and how identical insulators in particular can charge one another at all.

In this letter, we investigate the mechanism by which charges can be acquired and amplified by contact between

identical insulators in two independent experiments. Unexpectedly, we find that self-similar charge distributions seem to appear spontaneously on contacting surfaces, and we find evidence to suggest that this may occur through an instability by which localized charged regions beget further charges of the same sign. We find, furthermore, that insulators appear to charge at least in part by recruiting ions from the surrounding atmosphere.

Results. – To tribocharge similar insulators, we study a first model system that is easily reproduced: symmetric rubbing of common latex balloons as shown in fig. 1(a), and a second, more carefully controlled, laboratory system consisting of rolling of insulating disks in an instrumented apparatus sketched in fig. 2(a).

In the first system, balloons are obtained from the same lot, are the same color, are inflated to the same size, are initially washed with alcohol to remove surface oils or dirt, and are rubbed together symmetrically. As shown in fig. 1(a), the balloons spontaneously charge sufficiently to suspend one balloon from the other. A simpler experiment could scarcely be conceived, yet measurements (see fig. 1 caption) reveal that the balloons are initially charge-neutral, but following rubbing break symmetry to generate net charges, with one becoming highly positive (> 3 kV at 5 cm distance) the other equally negative. The choice of balloon polarity appears to be random: if we charge a pair of balloons and neutralize them by cleaning with alcohol, we find that, once dried and rubbed together again, the

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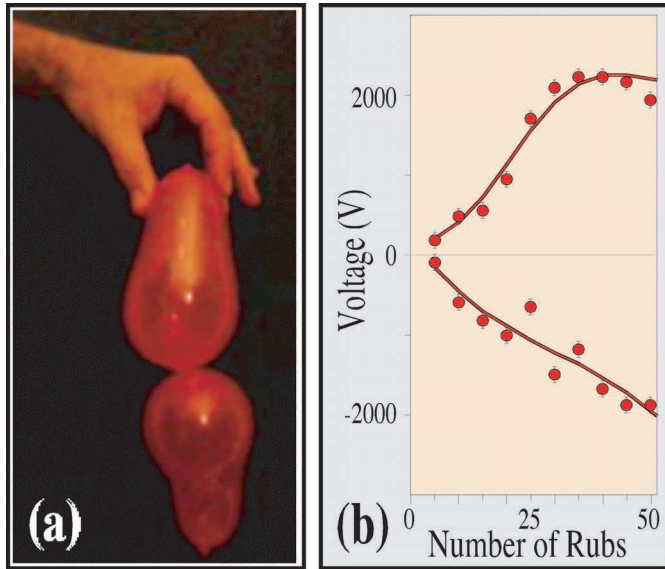


Fig. 1: (Color online) (a) Identical pair of latex balloons rubbed against one another adhere electrostatically; movies appear at <http://coewww.rutgers.edu/~shinbrot/save/TriboBalloons.html>. (b) The plot of voltage at a fixed position *vs.* the number of times the balloons are rubbed against one another shows that one balloon becomes positive and the other negative. One “rub” consists of rotating the balloons back and forth once while pressing the balloons together firmly; distances are measured from the closest point on the balloon to the probe tip. Voltages are measured using a null probe (Trek Inc., Model 344 with probe 6000B-7C, Medina, NY) attached to a fixed stand distant from other charges and conductors. Voltages and distances are recorded simultaneously using a video camera that records the voltmeter display and the balloon’s position against a backdrop containing a ruled measure.

same balloons produce new charges that show no apparent correlation to their prior polarities.

We emphasize that a balloon, once charged, has never been observed to become neutral or change sign with repeated rubbing. This is illustrated in fig. 1(b), where we show the voltage on negative and positive balloons, from separate experiments, after subsequent symmetric rubs against an identical balloon. The voltage is measured with a probe (see figure caption) placed 10 cm from the closest point of the balloon along its centerline. Figure 1(b) shows the characteristic result, which we reiterate has been repeated numerous times, that once a balloon has acquired a charge of a given polarity, its charge increases (notwithstanding ubiquitous but small noisy variations [3]) with repeated rubs, but never changes sign.

A similar effect is observed in an independent experiment in which identical polycarbonate disks are rotated under stepper control (fig. 2(a)). The disks are 70 mm across and 5 mm thick, with edges rounded so that the disks meet at a point. The disks are held against one another with a constant force of about 1 N. The charge on each disk is measured using a separate electrostatic sensor

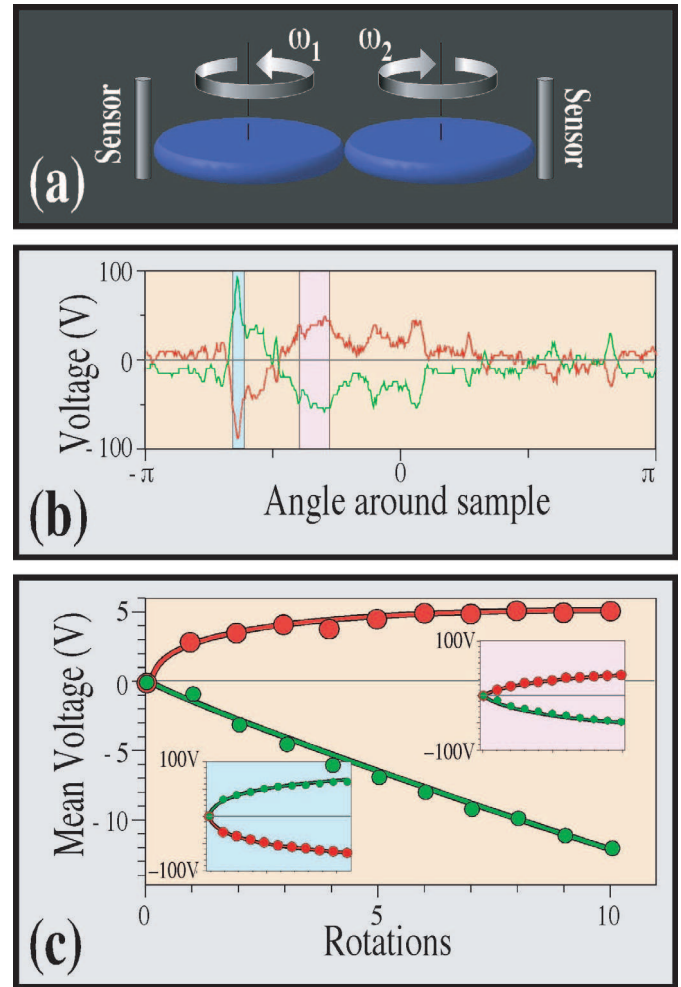


Fig. 2: (Color online) (a) Schematic of identical pair of polycarbonate disks rolling against one another under stepper motor control. The disk angular speeds, ω_1 and ω_2 are chosen so that the disks roll symmetrically: $\omega_1 = \omega_2$. Voltages are acquired at the surface of each disk using separate electrostatic sensors (see text). (b) Voltages measured by sensors *vs.* the angle around the sample, obtained here after 10 rotations. (c) Plot of averages of the circumferential voltage of each disk after each successive rotation. Insets show voltage averages of correspondingly colored regions of the disks identified in (b). In all cases, voltages consistently increase in magnitude, with one region spontaneously becoming positive and the other negative.

(Monroe Electronics, Lyndonville, NY) spaced about 2 mm from the disk surfaces at points opposite from the contact location. The disks are co-rotated at 0.24 rpm so that they roll against one another without rubbing, and are initially discharged by spraying with ionized air. By recording voltages as the disks rotate, we obtain a profile of voltage as a function of the angle around each sample, plotted in fig. 2(b) after 10 rotations. Spatial averages of the voltages around the circumferences of the two disks differ, as shown in the main plot of fig. 2(c) as a function of time. This plot shows a small but measurable difference, with each disk acquiring an increasing charge that, like the

balloons of fig. 1, steadily increases while maintaining its polarity. The difference in magnitude between positive and negative disks is close to the accuracy limit of the voltage probes, and is likely caused by simple instrument drift.

We note that because the two disks are of the same size and roll without slipping with the same speed, local regions come into repeated contact against one another, and so we can examine local, as well as global, charging of the disks. We do this by isolating two regions from the data record, one positive and one negative on each disk, as identified in fig. 2(b) in cyan and magenta rectangles. We show the averages of voltages within these regions as a function of time in correspondingly colored insets in fig. 2(c). Evidently locally as well as globally, charged regions consistently retain their polarity and increase their magnitude over time.

The results of these two experiments are surprising, both because it is unclear what provokes the symmetry breaking seen and because the very act of symmetry breaking works *against* the apparent electrostatic gradient imposed by Coulomb's law, which ought to oppose the motion of positive (negative) charges to the more electro-positive (-negative) surface. To analyze the effect, we next quantify the spatial and temporal distributions of acquired charges, which we do in three ways.

First, using the balloon model, we evaluate the moments of the charge distribution as a function of the distance from each balloon and the number of times the balloons have been rubbed together. For this purpose, we recall that the voltage is expected to decrease as a_n/r^n , for a multipole of moment n , where a_n is the magnitude of the n -pole's contribution and r is the distance to the center of the charge distribution [10]. Therefore we measure the voltage as a function of the distance from the charged balloon and attempt to fit the result to a power series with the goal of evaluating the multipole strengths, a_n . To make these measurements, we attach a null voltmeter probe to a fixed stand distant from other charges or conductors and slowly move the balloon toward the probe, recording the position and probe voltage with a video camera.

Against expectation, the voltage does not decrease as a limited series of powers, as it would if the charges were distributed as a net charge plus a finite set of multipoles. Instead, as shown in fig. 3(a), the voltage decreases exponentially to a high degree of precision: fits of the data shown to an exponential function have correlation coefficients between 0.998 and 0.999. This implies that the charge is not dominated by a small set of multipoles, but rather appears to be consistent with a charge distribution containing multipoles at all length scales. Moreover, although the charge saturates after about 25 rubbings, the distribution continues to produce charges at all scales during the charging phase as well as after saturation.

Second, we note that the measurement of the average voltage *vs.* the distance represents only an indirect evaluation of the charge distribution. To analyze the statistics

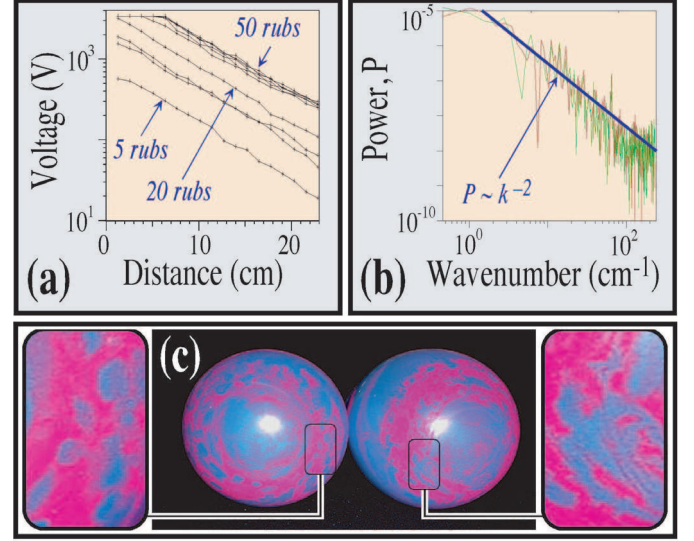


Fig. 3: (Color online) (a) Measurements of voltage as a function of the distance from a positively charged balloon *vs.* the distance from balloon to probe and the number of times the balloons are rubbed against one another. Data from a negatively charged balloon (not shown) are similar except for the sign. One “rub” in these measurements consists of rotating the balloons back and forth once while pressing the balloons together firmly; distances are measured from the closest point on the balloon to the probe tip. (b) Power spectrum of voltages shown in fig. 2(b) (color coded to agree with that figure), indicating a scale-invariant charge distribution up to a wavelength of about 1 cm. (c) Two latex balloons rubbed against one another, in this case in a symmetrical circular motion, and exposed to charged magenta toner. The toner is positively charged, so locations where it sticks are negatively charged. Notice that the left balloon’s charge patterns are dominated by elliptical spots (leftmost enlargement), while the right balloon is dominated by streaks (rightmost enlargement). The balloons used are “helium quality” natural latex rubber (Unique Industries, Philadelphia, PA), and are prepared by inflating with a hand pump and cleaning with ethanol and paper towels. The balloons are suspended by the nipple end to be dried. Once the balloons have dried, we hold the nipple ends of each balloon, and rub the opposite ends of the balloons together in a symmetric, circular motion. For voltage data, we use white iridescent balloons; for figs. 1(a) and 3(b), we use colored balloons for visual contrast.

of the charge distributions more directly, we evaluate the power spectrum of charges in the disk experiment, which we display in fig. 3(b) from voltage data obtained at the conclusion of the disk rolling experiment. This spectrum seems to obey power law scaling up to a distance of about $\lambda_c = 1$ cm, indicating that beyond containing multipoles spanning all scales, the distribution of charges on the disks is in fact self-similar [11] up to λ_c . Above λ_c , the power spectrum appears to saturate, suggesting that whatever mechanism is at work has a limited range given by λ_c . This may be because our disk size is close to λ_c , or there may be another mechanism leading to a characteristic wavelength: we cannot say at this point.

Third, to better visualize the spatial distribution of charges, we expose balloons to a cloud of charged toner, which adheres to oppositely charged regions on the balloons. In fig. 3(c), we have cleaned two balloons with alcohol and after air-drying we have rubbed the balloons together in a symmetric circular motion through 30 revolutions. Thereafter, we have exposed the balloons to a cloud of magenta toner (Samsung CLP-M300A) that is generated by shaking a small quantity of toner in a metal can beneath each balloon; shaking causes the toner to rub against the can, thus charging the toner. Measurements using the voltage probe indicate that the toner becomes positively charged, so the regions of the balloon to which toner adheres are negative. Two distinct patterns are boxed in fig. 3(c): elliptical spots on the balloon on the left, and striations on the balloon on the right. These patterns appear reproducibly, with the spots tending to form on balloons that are net positively charged, and striations on balloons that are negatively charged.

Finer details of these patterns can be revealed by exposing the balloons to “bipolar toner [12]” consisting of a blend of two different types of toner (Samsung CLP-M300A mixed in equal parts with Xerox 6R881): when these toners rub against one another, they charge oppositely and so can be used to identify both positive and negative charges on a surface. Characteristic results are shown for two balloons, one positive (fig. 4(a)) and one negative (fig. 4(b)). We find that spotted and striated patterns are often intermixed, with a single balloon exhibiting both types of patterns (cf. fig. 3(c)). While we cannot discount the possibility that these patterns—and the charging phenomena in general—are affected by manufacturing variations between the balloons, typically one pattern or the other will dominate on a single balloon, and the choice of pattern on the same balloon can change seemingly arbitrarily following cleaning. This leads us to believe that an intrinsic charging instability may be at work, with the spot pattern being characteristic of net positive charge and the branched striations of net negative charge. Moreover, since both spots and branches can be associated in other contexts with self-similar, recursive processes [13], we seek to determine whether the process underlying these unanticipated patterns may likewise be recursive.

To achieve this in a more controlled setting than simple rubbing of common balloons, we turn to simulations in which all details can be prescribed. To construct this simulation, we note from fig. 4(a) that the centers of the charged spots are not symmetric, but are eccentrically placed toward about 11 o’clock, while at 4 and 5 o’clock, smaller spots seem to radiate away from the lower spot. This suggests that the toner-identified charges may have been dragged by rubbing past fixed spots from 11:00 toward 5:00. We hypothesize that rubbing may by some mechanism cause smaller like-charged spots to be shed downstream: such a mechanism applied to ever-smaller spots is recursive and could be a candidate mechanism to

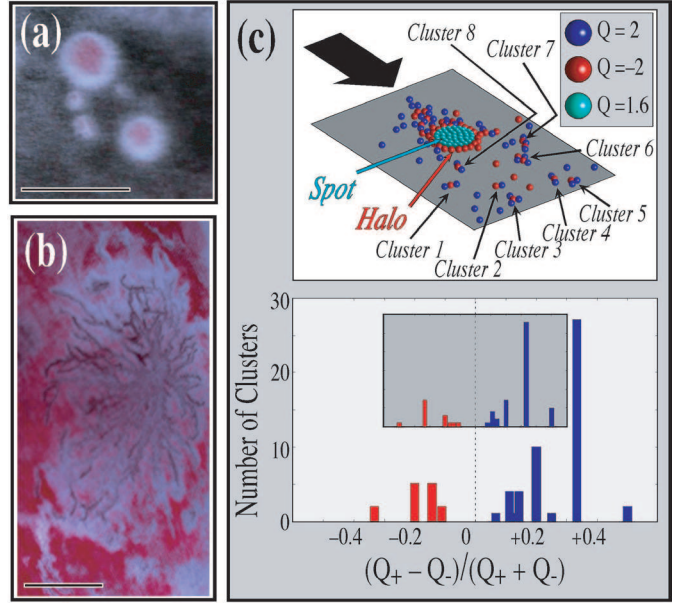


Fig. 4: (Color online) (a) Enlarged view of a spot pattern on a positively charged balloon. (b) Enlarged view of a branched pattern on a negatively charged balloon. Balloons are from a fresh pair that are cleaned, then rubbed against one another, and exposed to “bipolar” toner. Length bars are approximately 1 cm. (c) Simulations of the formation of charged clusters downstream of a charged spot. Upper panel: positive (blue) and negative (red) charged particles advected past a fixed charged spot (cyan) in the direction of the black arrow. Clusters identified are all positively charged except for cluster 2, which is neutral. Lower panel: distributions of charged clusters convected downstream in simulations of 250000 timesteps; blue bars are positively charged, red bars are negative. Main plot: distribution seen using a fixed timestep, Δt , where cluster charges are determined when clusters reach 20 particle diameters downstream of the fixed spot. Inset plot: distribution using a timestep of $2\Delta t$ where cluster charges are evaluated every 5000 timesteps.

generate a self-similar charge distribution. This scenario is consistent with gas discharge luminescence experiments in which charges trail downstream of the contact between two materials [14].

We apply a particle dynamics simulation to the problem of a fixed spot of charged particles. The simulation is performed in 2D, meant to mimic the material surface, and charged particles, representing smaller free surface ions, are accelerated with uniform force per unit mass, representing a steady force due to rubbing, past the fixed spot. Details of particle dynamics methods appear elsewhere [15]: in synopsis this model uses identically sized, elasto-plastic ions with kinetic sliding friction, and permits ions to interact electrostatically with inverse r^2 force as if their entire assigned charge were localized at the ions’ centers. Ions are randomly placed upstream of the spot with charges chosen to be either positive or negative with equal probability. We reason that if simple balloons exhibit charge amplification, then the effect may

be robust, and so the charges used in the simulation are somewhat arbitrary (see fig. 4(c) legend): we have varied both spot and ion charges by a factor of 2 and obtained similar results.

The configuration of charges is shown in fig. 4(c) (upper panel) after 60000 timesteps, where one new particle is generated upstream every 500 timesteps. The direction of constant acceleration of the ions is indicated by the large arrow, and the fixed charged spot is shown in cyan. A red halo of opposite charges forms spontaneously and remains throughout the simulations, although individual ions in the halo become washed downstream by the imposed acceleration. We propose that the halo may paradoxically attract positive particles (blue), causing these ions to be shed downstream as shown in the figure, while negative ions (red) are repelled and travel as lone charges.

We record the positions of all ions every 5000 timesteps for later analysis. From this data record, we evaluate how many clusters of 2 or more ions are positively and negatively charged using a positively charged fixed spot. We are interested in net charging, and so neutral clusters (which are plentiful) are ignored. Figure 4(c) (upper panel) shows 8 clusters, of which all but cluster 2 are positively charged. This example is fortuitous: at most times, somewhat fewer clusters are positively charged. The numbers and charges of clusters are evaluated in fig. 4(c) (lower panel), which shows the total charges on clusters downstream of the fixed spot from two simulations of 250000 timesteps. The clusters evolve over time, and so we evaluate cluster charges in two ways. In the main plot we show cluster charges evaluated when a cluster first reaches a “finish line” 20 particle diameters downstream of the spot center, while in the inset (using the same axes), we evaluate the charge on every cluster every 5000 timesteps. Thus in the second case, we count clusters more than once as they travel downstream; furthermore to confirm that these results are not mere computational artifacts, we double the timestep in the simulation shown in the inset (and we therefore spawn new upstream ions twice as often: every 250 timesteps). In both cases we obtain similar results: in the main plot we find that 50/64 clusters are positively charged and in the inset we find 49/63 clusters are positive.

These simulations suggest that charged spots shed smaller charged clusters that are predominantly (about 75%) of the same sign. Since charge is conserved, it follows that oppositely charged ions must be unattached. It is observed in experiments that balloons emit a crackling sound during rubbing [12], and it seems likely that this may be caused by the discharge of isolated charges to the opposing balloon, which leave less mobile clusters of charges attached to the originating balloon. This conjecture remains for future investigation, as does the natural speculation that the production of smaller spots from larger ones—as well as discharge patterns formed following discharges—may both produce the measured self-similar charge distributions.

Discussion. — We have shown in two examples—first using common latex balloons rubbed by hand, and second using instrumented polycarbonate disks rotated with stepper motors—that similar materials can tribocharge when symmetrically rubbed against one another, despite lacking either an obvious source for charge carriers or an apparent triboelectric gradient to drive the charge transfer. We have found both by quantitative measurement of voltage as a function of distance and by qualitative examination of the patterns on charged balloons that the charges that accumulate appear to be complex and may be consistent with a recursive, self-similar, model for charge formation, a result supported by power spectral analysis of the disk experiment. We have proposed and confirmed by direct simulation that once a fixed spot of charge forms on an insulating surface, an oppositely charged halo appears to catalyze the downstream formation of smaller clusters of similar charge to the original spot. We have conjectured that this may be the source of the positive self-similar charge distributions seen. The negative charge distributions (fig. 4(b)) could in this scenario be Lichtenberg discharge patterns formed by the discharge of the free negative ions produced by the model (fig. 4(c)).

Although only speculative at this stage, the hypothesis that positive spots generate more positive spots and liberate free negative ions appears to explain many of the observations seen in natural and laboratory tribocharging. In particular, this hypothesis suggests that the mechanism of triboelectric charging may be intrinsically unstable, so that very small variations in surface charges, geometry or chemistry may tend to become amplified. Thus, a single charge on a surface may become an engine for the generation of like charges. The notion that charges may generate more charges of the same sign is counterintuitive, however the phenomena that we seek to understand are themselves inexplicable without some such mechanism, and we propose that this first hypothesis may serve as a catalyst for more rigorous testing.

Open questions. — The results that we have presented are easily reproduced, but leave much to be understood. First, as we have remarked, the simulation described is intended as a candidate to promote further study, and is certainly not definitive. For example, the mechanism proposed should operate equally on both contacting surfaces, and so how two surfaces compete to determine which will become positive and which negative is not resolved by this simulation and must await future study.

Second, it is not known how insulators recruit the free charges that manifestly must be present during tribocharging. The answer to this, very basic, question remains elusive, however to inform future investigation, we have performed a final set of experiments to establish the extent to which the charges may be recruited from free ions in the air. The species of airborne ions have been identified elsewhere [16], and are found to include higher-mobility anions, such as O^- and O_3^- , alongside

lower-mobility cations such as $(\text{NO}_x^+)(\text{H}_2\text{O})_n$. It seems plausible that the lower-mobility cations may contribute to predominantly positive spots (fig. 4(a)) through a mechanism such as that described in fig. 4(c), while predominantly negatively charged striations (fig. 4(b)) may be produced by Lichtenberg discharges of higher-mobility anions. We note, however, that there is in principle no reason that the simulation of fig. 4 would not work equally well for negatively charged spots as for positively charged ones, and the question of why spots tend to be positive remains open.

To investigate this issue further, we inflated identical balloons using helium, which is inert and lacks these identified species, and we rubbed these balloons together in air and in an atmosphere of the inert gas. We found in multiple separate trials that balloons readily charge if filled with inert gas and rubbed in the presence of air, however the same balloons filled either with air or with inert gas do not tribocharge when rubbed against one another in an inert atmosphere. Triboelectrification of dissimilar materials is not strongly affected by the surrounding gas, which we have confirmed in experiments in which we find that the balloons in any atmosphere continue to be readily charged by rubbing with paper or cotton. The presence or absence of triboelectrification was established by determining whether balloons attract one another and was confirmed by direct measurements using the voltmeter probe. These experiments were performed in an inverted 15 gallon tank that traps the helium. The lack of tribocharging between identical materials in an inert atmosphere suggests that the recruitment of free charges by insulators occurs either as a result of the air itself (*e.g.* following dissociation of air ions during discharges) or by alteration of polymeric surface states by air molecules. In either case, it appears that the interaction between the surface and air plays a significant, but yet to be clarified, role in the tribocharging instabilities that we have studied.

In conclusion, we find that similar materials, contacted symmetrically, can charge one another. We caution that the effect that we report is much weaker than that seen in tribocharging of dissimilar materials. Moreover, the disks described in fig. 2 were worn from use in multiple experiments, both for symmetric ($\omega_1 = \omega_2$) and non-symmetric ($|\omega_1| \neq |\omega_2|$) protocols [6]. As with any realistic material, the possibility that history plays a role in charging cannot be discounted, and although the disks were of identical stock and were discharged prior to experimentation, it is possible that the voltages acquired are associated with surface impurities or wear variations. We presume that completely identical and virgin materials prepared in exactly the same way and contacted symmetrically without fluctuations or contamination would have no basis to choose to break symmetry in one direction or the other, and would remain charge neutral. Nevertheless, the effects that we report seem to be reproducible using simple methods and materials, and our data indicate that realistic materials under conditions

that are identical for most practical purposes exhibit an instability that amplifies any incipient localized charges. As we have mentioned, the polarity of charges acquired in the balloon experiments appears to be uncorrelated with the polarity produced using the same pair of balloons in preceding experiments, which again is consistent with an intrinsic instability. Future detailed analysis of surface composition and morphology at regions of positive and negative charging would be valuable. For the present, what factors govern the symmetry breaking seen in either of our experiments, or precisely what role history, wear or contamination play remains mysterious, as it has since the first charging studies a half-millennium ago. What seems evident is that similar materials under practical circumstances acquire charge through an as-yet unexplained mechanism that merits more thorough study.

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