Granular Coarsening

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The phenomenon of coarsening has been the topic of intensive study, chiefly in material science, where droplets coalesce[1,2] or successive layers of material deposited on a substrate form complex morpholgies[3,4]. These problems are of technological interest because properties of composite materials such as polymer blends and alloys depend on the size and structure of included droplets[5,6]; likewise deposited layers are used in applications ranging from semiconductor design to catalysis[7,8]. In this letter we examine coarsening in a different context: granular flow. If a shallow bed of fine grains is shaken at moderate amplitude and in the presence of air, heaps develop that coarsen over time[9,10]. We find experimentally that this coarsening scales as a power law over time with exponent 1/3, and we present a simplified model and direct analysis that scale with the same exponent.

Heaping

In Fig. 1(a)-(d), we show top views of a bed of 250μ glass beads. The bed is 20 particles deep on average, and is shaken nearly sinusoidally with amplitude A = 0.94 cm and acceleration Γ = 1.3 g. Heaps form spontaneously in our experiments for amplitudes larger than about 0.5 cm and for accelerations above 1.2 g. The locations and sizes of heaps are governed initially by random arrangements of irregularities in the granular layer. Once heaps are formed, they coarsen to form larger heaps. This is shown in successive snapshots taken 45 seconds apart in Fig's 1(a)-(d). From direct visual observation we find that particles flow continuously down the heaps into valleys, where, by continuity, they must be re-entrained into the heaps. Moreover, definitive experiments performed elsewhere show that heaping vanishes if either the pressure in the container is reduced below about 20 Torr or if the container bottom is made porous[11].

The mechanism for heaping is thus believed to be that particles roll down the free surface of the heap and are then re-entrained, apparently by air flow produced when the granular bed separates from the container during its downstroke. At its most basic level, the instability leading to heaping can be approximated using Darcy's law for flow through porous beds, as follows[12]. Consider a uniform granular bed resting on a flat, oscillating, surface. As the surface accelerates downward faster than gravity, a gap will necessarily develop between the bed and the surface, and air will flow through the bed to fill the gap. Darcy's law provides that the air flow speed, S, through a region of the bed of area A and thickness

h is given by $S = c_1 Ap/h$, where c_1 defines the material permeability, and p is the pressure differential across the bed. The heaping instability can occur when S becomes sufficiently large that drag on small particles exceeds their weight, and they become entrained into the airflow. When this occurs, the thickness of a depression in the bed can decrease at a rate proportional to the air flow speed. In particular, a depression starting from an initial thickness h_o will lose material at a rate $c_2 S$ for some constant c_2, so a depression of fixed area will diminish in thickness according to: $h(t) \quad h_o - c_2 St$. Inserting the Darcy approximation for S gives to leading order: $h \quad \left(1 + \sqrt{1 - c_3 t}\right) h_o/2$ where $c_3 = (4 c_1 c_2 A p)/h_o^2$.

This relation is not exact in this transient and complex problem -- where for example dilation causes the bed permeability to vary spatiotemporally[11]. Nevertheless this approximate analysis seems to reveal the root mechanism believed to be at work, namely that rapid air flow in regions of diminished depth entrains small particles, and this in turn further reduces the depth in that region. This instability grows until it saturates in finite time, at $t = 1/c_3$.

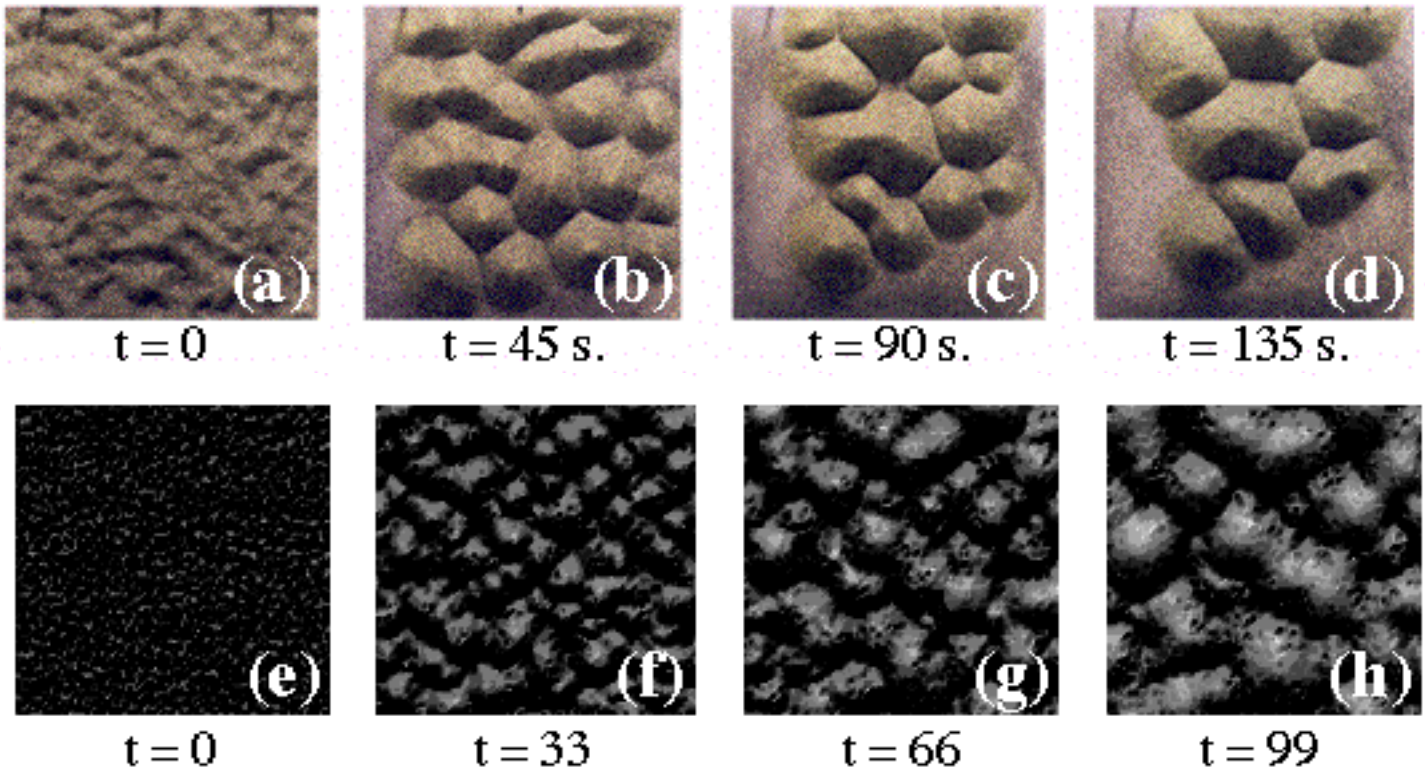

Figure 1 - Top view of coarsening of granular bed in experiment (above) and simulation (below). In both cases, hills are formed spontaneously starting from randomized initial states at t = 0, and these hills coarsen over time as smaller hills are engrossed by larger ones.

To study the effect in greater detail, we consider the following phenomenological model. We have mentioned that the granular motion in this problem appears to consist of two parts: flow down the surface of hills, and particle entrainment near valleys. We model these separately. To model flow down a hill we observe that for high dissipation (characteristic of collective particle flow[13]), particles rapidly assume their terminal velocity. It is known, moreover, that the thickness of a shallow flowing granular layer grows with its maximum speed[14,15].

Thus we perform the following simulation. We define the bed height to be a continuous function, $h(\mathbf{r})$, on a grid of $N \times N$ horizontal points $\mathbf{r} = (x,y)$, where x, and y are integers ranging between 1 and N. Boundary conditions are periodic. At all grid locations, we apply three computational operations.

First, we compute the local gradient, ∇h at all grid locations, $\mathbf{r}$. The local gradient is calculated separately in $\hat{x}$ and $\hat{y}$ directions, and is defined to be the slope between h at the given gridpoint, $\mathbf{r}$, and its nearest *downhill* neighbor in either the $\hat{x}$ or $\hat{y}$ direction. If there are two downhill neighbors (i.e. $h(\mathbf{r})$ lies at the crest of a hill), then the slope is defined to be the steepest downhill direction. If $h(\mathbf{r})$ is equally steep in both opposing directions, or if $h(\mathbf{r})$ lies in a valley, then the sign of the slope is chosen at random. This final detail is crucial: we will see that this randomization is what leads to coarsening in this problem.

Second, we move material from hills to valleys at a rate defined by the gradient. Specifically, we decrease $h(\mathbf{r})$ by an amount proportional to ∇h, and increase $h(\mathbf{r}')$ by an equal amount, where $\mathbf{r}'$ is a gridpoint located a distance (defined below) from $\mathbf{r}$ which also depends on ∇h. The term D_{rt} is included to account for diffusion, and is chosen at each grid location and time increment to be a pair of pseudo-random numbers on $[-\delta,\delta]$, where δ defines a magnitude for the diffusivity. This algorithm conserves global volume, i.e. $\int h(\rho)d\rho$, exactly because material removed from $\mathbf{r}$ is invariably deposited elsewhere, at $\mathbf{r}'$.

Third, we model particle entrainment near valleys by reversing the direction of the flow if a criterion for closeness to a valley is met. Explicitly, after a small timestep Δt, h evolves according to:

$$\begin{aligned} h(\mathbf{r}, t+\Delta t) &= h(\mathbf{r},t) - \alpha|\nabla h(\mathbf{r},t)| \\ h(\mathbf{r}', t+\Delta t) &= h(\mathbf{r}', t) + \alpha|\nabla h(\mathbf{r},t)| \end{aligned} \qquad [1]$$

where $\mathbf{r}' = \mathbf{r} \pm \beta \nabla h + D_{rt}$ for $|\nabla h| \lesseqgtr \gamma$, and where α, β, and γ are constants. Thus α determines the depth of material moved from one location to another in a timestep, β governs the distance, or speed, at which material is transported per timestep, γ establishes the strength of re-entrainment of material, and δ is a measure of diffusivity.

Typical results of this simulation are shown in Fig's 1(e)-(h). To aid in comparison with experiment, a top view of h(**r**) is shown using synthetic lighting. This simulation contains 100 × 100 horizontal grid points and starts from a pseudo-random initial state[16].

Scaling

Simulations in which points in valleys are *not* entrained at random do not exhibit coarsening. Thus the cause of coarsening appears to be that particles in valleys between heaps can be entrained under either adjacent heap. As a result, heaps wander and so accrete smaller neighboring heaps. We can derive the expected growth rate as follows.

Consider a heap of volume V_H, that sheds a particle displacing a volume δv. Given that particles in valleys are randomly entrained into adjacent heaps, we expect the volume of any particular heap to execute a random walk in magnitude as particles are absorbed from or shed onto neighboring heaps. After n particles are shed, the expected volume of a heap will thus obey:

$$\langle V_H \rangle = V_o + \delta v p \sqrt{n}, \qquad [2]$$

where V_o is an initial volume, and p is a probability defining the net rate of transport across an interface betwen adjacent heaps. The number of particles shed can be expected to increase linearly with both the volume of the heap, $\langle V_H \rangle$, and the time elapsed, t, so for small V_o one obtains:

$$\langle R_H \rangle = c_4 \left[2V_o + \left(c_5 p^2 v^2 \right) t \right]^{1/3}, \qquad [3]$$

where $\langle R_H \rangle$ is the mean heap radius and c_4 and c_5 are constants. In contrast with other, e.g. spinodal, examples of coarsening, the universal exponent 1/3 is independent of balances between inertia and drag or of material characteristics such as surface tension or viscosity.

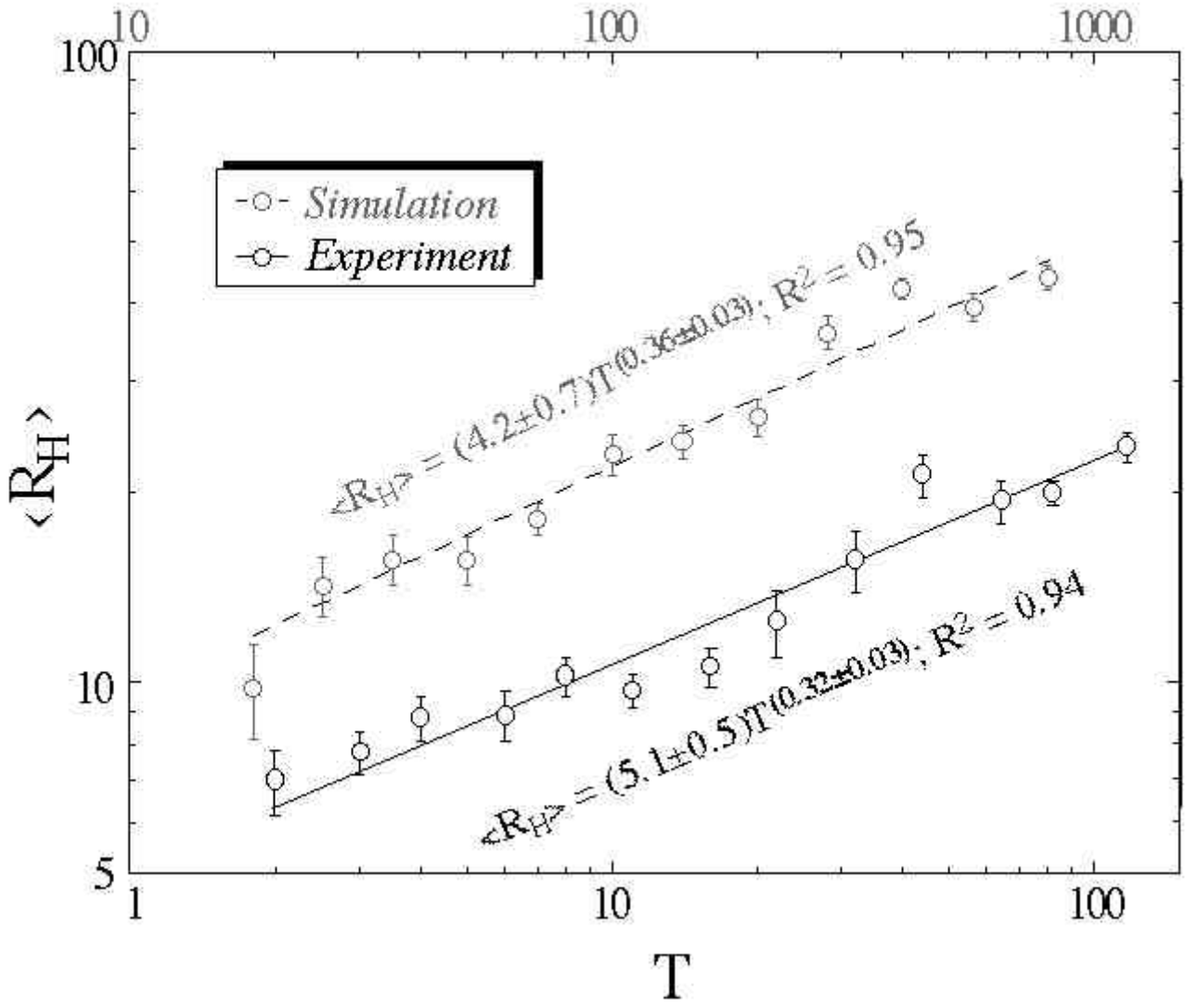

Figure 2 - Scaling of heaps in experiment and simulation

In Fig. 2, we plot the scaling of mean heap radius with time from both experiment and simulation, giving exponents 0.33 ± 0.03 and 0.36 ± 0.03 respectively. $\langle R_H \rangle$ is directly measured in both cases by superimposing an orthogonal lattice on shapshots of the heap viewed from above, and measuring the distances between successive valleys along lines of this lattice. Time is expressed in seconds in the experiment, and in number of computational steps in the simulation.

Conclusion

We note four things in closing. First, quasi-1D granular coarsening has recently been observed in careful experiments of grains trapped in a vertically shaken annulus between two vertical cylinders[9]. In those experiments, the coarsening was determined to grow exponentially with time, implying the existence of a characteristic time scale. In our quasi-

2D experiments, we find a power law scaling, without an evident timescale. This is curious and unexplained.

Second, prior models, based for example on droplet coalescence[1,17] or on variations in effective coefficients of restitution of particles near heaps[10] rely on knowledge of material particulars, for example viscosities, surface tensions, or elastic properties. The simplified model examined here does not depend on these particulars, but nevertheless produces the universal scaling exponent 1/3. This result may lend support the postulate that coarsening in granular beds may have correlates with coarsening in other problems.

Third, although both experiment and model exhibit the anticipated universal scaling, several features particular to this experiment remain to be explained. For example, valleys between heaps tend to meet at close to 120° angles; this is particularly evident in Fig. 1(d). This is not reproduced in the simulations, based as they are on an oversimplified model which neglects all details of fluid-grain interactions. Similarly, in our experiments, fingers are seen around isolated boundaries of heaps.

Finally, by modifying the four parameters α, β, γ and δ -- corresponding respectively to volumetric flow rate, transport speed, re-entrainment fraction and diffusivity -- we can obtain predictions that may be useful outside of the context of granular flow. We find that coarsening grows with $t^{1/3}$ in simulations that vary each of these parameters by factors ranging from 2 to 10. Nevertheless, the surface roughness -- defined to be the rms fluctuation in height -- can be altered with changes in parameters. For example, by increasing δ by a factor of 10, although coarsening proceeds at the same rate as before, the surface roughness after 99 timesteps is decreased by a factor of nearly 4 over the value found from Fig. 1(h). This suggests that surface roughness in other problems may be decreased by increasing the diffusivity (or temperature), but that this will not change the ultimate surface morphology[3]. To take a second example, by increasing β by a factor of 10, the surface roughness is decreased by a factor of nearly 6, again after 99 timesteps as compared with Fig. 1(h). The surface morphology in this instance, however, becomes much more irregular, more closely resembling a cratered appearance than the previous hilly character. These results suggest that annealing material surfaces (thus increasing diffusivity) can reduce roughness while retaining the surface morphology[6,18,19], whereas increasing the rate of material deposition may more rapidly decrease the surface roughness, while significantly altering its morphology.

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16 The initial state is a set of 100 × 100 pseudo-random numbers uniformly distributed on (0,R), where R is chosen to be 30. Provided that R ‹ 100, no noticeable differences in coarsening behaviors are seen. For R ›› 100, V_o in Eq. [3] (following) becomes significant, and non-universal scaling is observed.

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