Granular Matter 2, 65–69 © Springer-Verlag 2000

Harmonic patterns in fine granular vibrated beds

T. Shinbrot, L. Lomelo, F. J. Muzzio

Abstract We examine the spontaneous formation of numerous previously unreported patterns in deep beds of fine grains vibrated at frequencies ranging from 10 to 100 Hz and accelerations ranging from 1 to 14 times gravity. Similarly to shallow beds, we find stripes, closed cells and labyrinthine patterns. In addition, we observe traveling waves, 'cannibalizing' cells and other behaviors that are unexpected and remain unexplained. All of these patterns vary spatiotemporally, and unlike shallow bed patterns, are harmonic rather than subharmonic.

Experiments in shallow vibrated beds of grains are known to produce a rich variety of subharmonic granular patterns, including stripes, squares, hexagons, and solitary structures [1]. In deeper beds, on the other hand, considerably less is known, and experiments reveal complex and incompletely understood inter-relationships between the dynamics of the grains and those of the interstitial fluid [2, 3]. Nevertheless, deep vibrated beds are of widespread industrial interest since they are commonly encountered in practical problems ranging from trucking of granular goods to vibratory sieving to promoting flow from hoppers by the use of impact hammers. In this letter, we briefly survey pattern formation at the surface of deep, vertically vibrated, beds of fine grains [2–4, 9]. We catalog a rich variety of new patterns and we report on transitions between these patterns. All of the patterns reported here vary spatiotemporally, and (unlike patterns in shallow beds) are harmonic in that they change little from one cycle to the next.

Our experiment consists of a 6 cm deep bed of fine spherical glass beads, vibrated in the presence of air. The beads are polydisperse in the range 60–180 µ. The patterns reported appear also in comparison experiments using both 3 cm and 12 cm deep beds, but are *not* seen with larger (e.g. 1000 µ) beads. The container was vibrated over a span in frequencies, *f*, from 10 to 100 Hz, with maximum accelerations ranging from $\Gamma = 1$ to 14 times gravity. Vibration was generated using an electromechanical shaker (Vibration Test Systems, Aurora, OH). The shaker was supplied with a sinusoidal voltage signal, but the actual shaker oscillation is known to be different from sinusoidal due to the delayed response of the granular bed [5]. The maximum acceleration was measured using an piezoelectric accelerometer (PCB Piezoelectronics, Depew, NY).

The patterns that we report can be divided naturally into two sets based on the mechanisms that appear to dominate the flow. Below about 45 Hz, the bed develops a surface sloped at between 20° and 28° to the horizontal, while above this frequency the bed exhibits a transition to a nearly flat surface. It has been demonstrated previously that the mechanism responsible for maintaining this slope is circulating air flow drawn into the bed by the down stroke of the container at accelerations faster than the bed can respond [2]. Thus we discuss in turn vibrations from about 10 to 45 Hz, where the flow is dominated by this air circulation, and vibrations from about 45 to 100 Hz, where more complex interactions appear to be at work. We summarize the parametric dependence within these frequency ranges in Figures 1 and 3 respectively, and in Figures 2 and 4 we display corresponding snapshots of the surface patterns observed.

The lower frequency range can be divided into three qualitatively different subranges. We begin with a discussion of the lowest subrange, from 12 to 15 Hz. As indicated in Fig. 1, three distinct patterns can be identified in this subrange, each of which is superimposed on the spontaneously sloped surface. Flow here appears to be cohesion dominated: at the lowest accelerations, around $\Gamma = 1.6$, particles begin to flow only intermittently as their hold on the supporting bed is overcome, and layers of grains a few centimeters in diameter and under a millimeter in depth slowly slide – nearly intact – down the slope. We term this behavior 'sheeting', and display a snapshot of the state in Fig. 2(a). All snapshots are illuminated from the side to accentuate the relief of the patterns, and the scale of all snapshots is identical. Away from the sliding regions, the surface is nearly quiescent, disturbed only by small numbers of grains bouncing down the slope. At slightly higher accelerations, sheeting coexists with a second pattern: small regions of agitated grains, under a half centimeter wide in the spanwise direction and between 1 and 2 centimeters long in the streamwise direction, climb up from the bottom of the slope. The mechanism for this paradoxical motion has been discussed previously [6] and is due to material from above the cell accreting onto its leading edge more rapidly than it is removed from its trailing edge. These climbing cells are eventually met and

Received: 17 May 1999

T. Shinbrot (✉), L. Lomelo, F. J. Muzzio,
Dept. of Chemical & Biochemical Engineering,
Rutgers University,
Piscataway, NJ 08854, USA
e-mail: shinbrot@sol.rutgers.edu, muzzio@sol.rutgers.edu

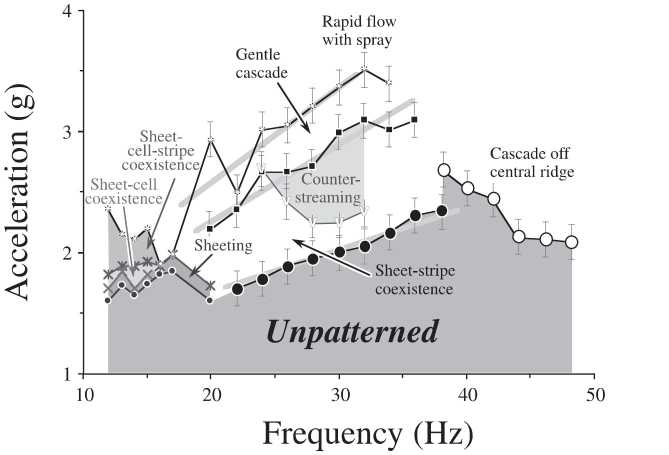

Fig. 1. Parametric dependence of surface patterns seen in deep bed of fine glass beads vertically vibrated at frequencies under 50 Hz. All data are produced by fixing a frequency at a low acceleration and slowly *increasing* the acceleration until the recorded transitions are observed. Most transitions appear to be hysteretic, and transition values produced by *decreasing* the acceleration are not reported here. Error bars represent estimated uncertainty associated with the bed response, as discussed in text. Grey lines are at constant Froude number, as discussed in text

annihilated by sheeting of grains sliding downward from the top of the slope. At higher accelerations, stripes appear at the region where the climbing cells meet the falling sheets; thus in the parameter range indicated in Fig. 1, all three patterns coexist at different locations along the slope. The stripes consist of thin, streamwise-aligned rivulets of particles, as identified in Fig. 2(a).

At higher accelerations yet, above about $\Gamma = 2.2$, the apparent cohesive integrity of the bed is broken, and clearly defined patterns give way to a vigorous downward flow. In this 'rapid flow with spray' state, shown in Fig. 2(b), the cascading grains appear to not have time to equilibrate with the supporting bed before they are thrown back into the air by the energetic shaking of the container. The bed remains steeply sloped, and the abundant supply of rapidly cascading grains implies that there must be a vigorous granular recirculation loop beneath the surface [7].

In the next frequency subrange, above about $f = 20$ Hz, flow becomes continuous, rather than intermittent, and it becomes possible to determine rudimentary kinematic balances for the flow. The surface remains sloped at an angle of about 20° to the horizontal, and two new patterns emerge, as indicated in Fig. 1. At accelerations above about $\Gamma = 2$, there is coexistence between sheeting flow near the top of the slope and thin, streamwise

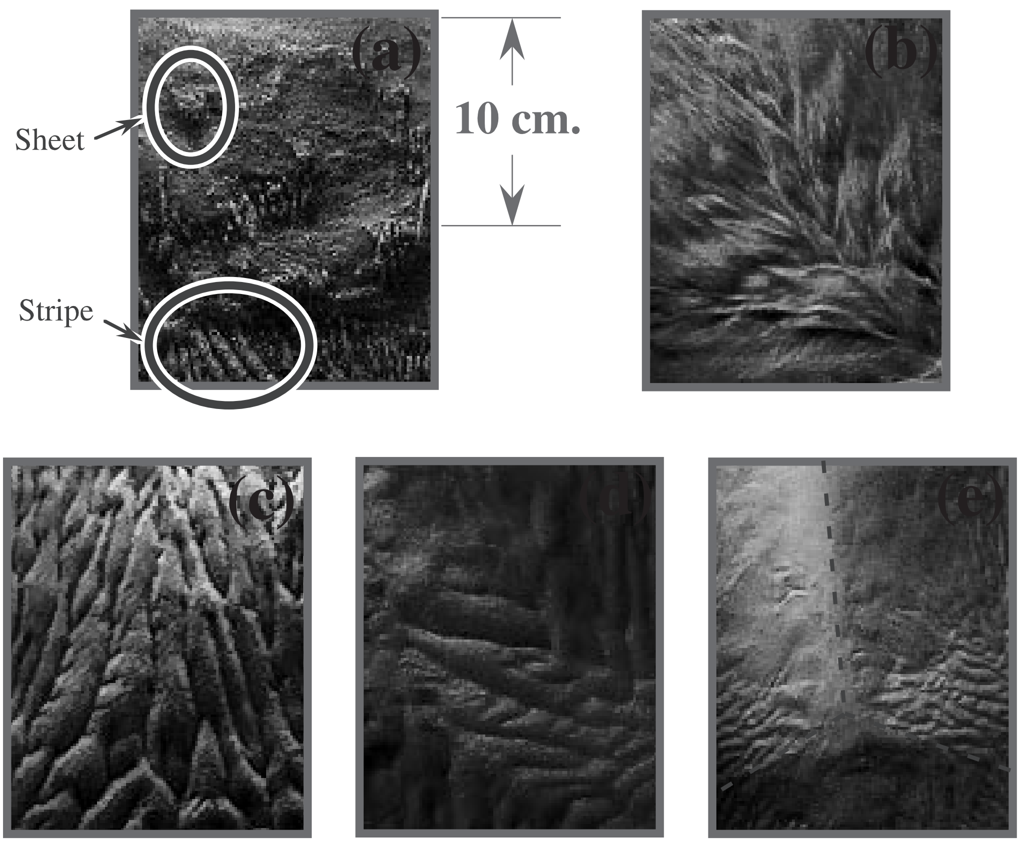

Fig. 2a–e. Snapshots of several states indicated in Fig. 1. States shown are referred in the text as **(a)** sheet-stripe co-existence, **(b)** rapid flow with spray, **(c)** counter-streaming, **(d)** gentle cascade and **(e)** cascade off central ridge. These photographs are taken from top views of the surface, illuminated from the side to enhance contrast. The downhill direction is toward the bottom right corner of each inset, and the scale in all photographs is identical

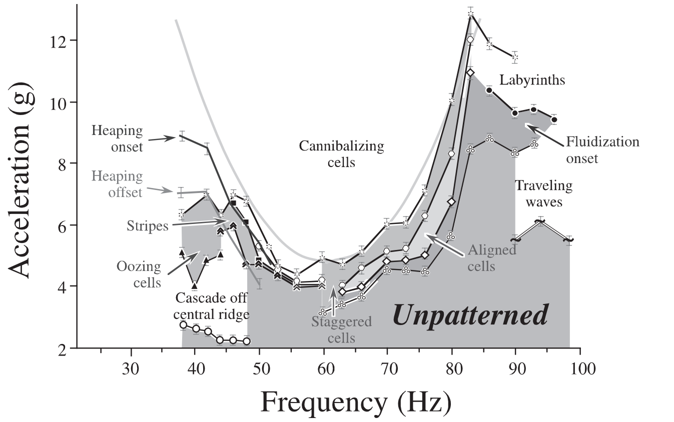

Fig. 3. Patterns seen in deep bed of fine glass beads vibrated at frequencies between 40 and 100 Hz. The surface is nearly flat in this frequency range. The grey curve defines a constant dynamical lengthscale, as discussed in text

oriented, stripes near the bottom, (Fig. 2(a)). At accelerations close to $\Gamma = 2.5$ g, an island (identified in Fig. 1) of 'counterstreaming' flow is observed in which both downhill and uphill motion of stripes is seen. The peaks of the stripes in this state, shown in Fig. 2(c), travel *downstream*, while the valleys appear to travel *upstream* [http://sol.rutgers.edu/∼shinbrot/DeepPatterns/Counterstreaming.mpg]. At higher accelerations, the coarsening and *streamwise*-aligned stripes abruptly vanish and are replaced by another pattern, a 'gentle cascade' flow, shown in Fig. 2(d), in which cells with constant separation, aligned roughly *spanwise*, creep slowly downstream by the ambient flow.

We can analyze the kinematics of this problem by writing the Froude number for flow down the slope:

$$Fr = \frac{\nu^2}{gd} \propto \frac{\Gamma^2 g}{f^2 d} \ , \tag{1}$$

where d is the height of the slope, and we have estimated the granular surface velocity to be proportional to the maximum vibration velocity, $\Gamma g/f$. From this lowest-order analysis, we expect to see hydrodynamic-like transitions along lines of constant *Fr*; i.e. constant Γ/f. In Fig. 1, we plot constant *Fr* lines in grey, passing through the lowest acceleration at which steady flow can conceivably persist, $\Gamma = 1$, $f = 0$.

In the final frequency subrange shown in Fig. 1, above about $f = 40$ Hz, there is a transition pattern in which the uniformly sloped surface gives way to a central, sharply ridged, heap (identified by broken lines in Fig. 2(e)) in the middle of the container. This transition pattern exhibits a flow of narrow stripes cascading off of the central heap. Our container is a cube, 18 cm. on a side, and it seems plausible that the location of this transition pattern may depend on container size. The precise location aside, this state marks a transition between a strongly sloped and a nearly flat bed, described next.

At frequencies between 40 and 100 Hz, the surface becomes nearly flat and a second set of patterns emerges, summarized in Fig. 3. The transition between sloped and flat beds is sharp and strongly hysteretic as indicated by onset and offset lines in Fig. 3, with a range between heaping onset (evaluated at increasing Γ) and offset (at decreasing Γ) acceleration of as much as $2g$. The presence of this transition provides practical implications for the mixing and segregation of granular materials – including the presence of a reversible segregating state – which have been discussed elsewhere [8]. As before, there are three distinct subranges, the first being from about 40 to 50 Hz. In this subrange, sharply ridged heaps form, suggesting that multiple counter-rotating convective rolls may be extruding grains from beneath the visible surface. Below about $\Gamma = 6$, numerous small (∼1 cm across) and well defined cells can be seen, as shown in Fig. 4(a). Each cell here maintains its individual identity for a period on the order of several seconds as it travels away from the nearest peak and toward the nearest valley. This gives the appearance of 'oozing cells' which continually exude from the peaked ridges of the heaps and creep toward the more rounded valleys [http://sol.rutgers.edu/∼shinbrot/DeepPatterns/Oozing.mpg]. At higher frequencies and accelerations, these cells give way to stripes, again superimposed on sharply ridged heaps as shown in Fig. 4(b). In this case the oozing flow is replaced by a current along the direction of the stripes, so there is less apparent motion in the surface pattern.

At higher accelerations yet, in this frequency subrange and at higher frequencies, 'cannibalizing cells' are found (Fig. 4(f) and Fig. 5(a)–(b)), which, unlike the oozing cells just described, change identity very rapidly, with individual cells losing integrity as they consume their neighbors, typically in under a second. These rapid changes in identity are accompanied – and perhaps caused – by significant fractions of surface particles becoming airborne, during which time they form a network of finely structured filaments between cells, as shown in Fig. 5(a)–(b). It is not known whether the onset of these patterns is dominated by air-particle interactions or by inter-particle effects, e.g. cohesion.

A second qualitatively different frequency subrange appears between about $f = 50$ and 80 Hz. As shown in Fig. 3, while cannibalizing cells are seen at the highest accelerations, three other states are found at lower accelerations. At the lowest acceleration, the surface becomes nearly uniformly fluidized, but no organized patterns are seen. Between these extremes in acceleration, two cellular patterns are found. For accelerations slightly above the fluidized surface state, one observes closed cells that tend to be aligned in rows, as shown in Fig. 4(c). At slightly higher accelerations, these aligned cells give way to staggered, nearly hexagonal, cellular arrays [9], displayed in Fig. 4(d). Although there is some overlap between these two cellular states, in the 'staggered cells' state, we find qualitatively fewer aligned cells than in the 'aligned cells' state.

All of the patterns seen in this subrange are cellular in morphology, with a well-defined characteristic diameter. By dimensional analysis, the lowest order length scale produced by driving an arbitrary system at acceleration Γg and frequency f is just $\lambda = \Gamma g/f^2$. In Fig. 3, we plot in grey a curve of constant λ. This curve necessarily has several free parameters, and is included only for the sake of comparison; nevertheless the conjecture that flow in this regime is governed by simple dynamical effects associated

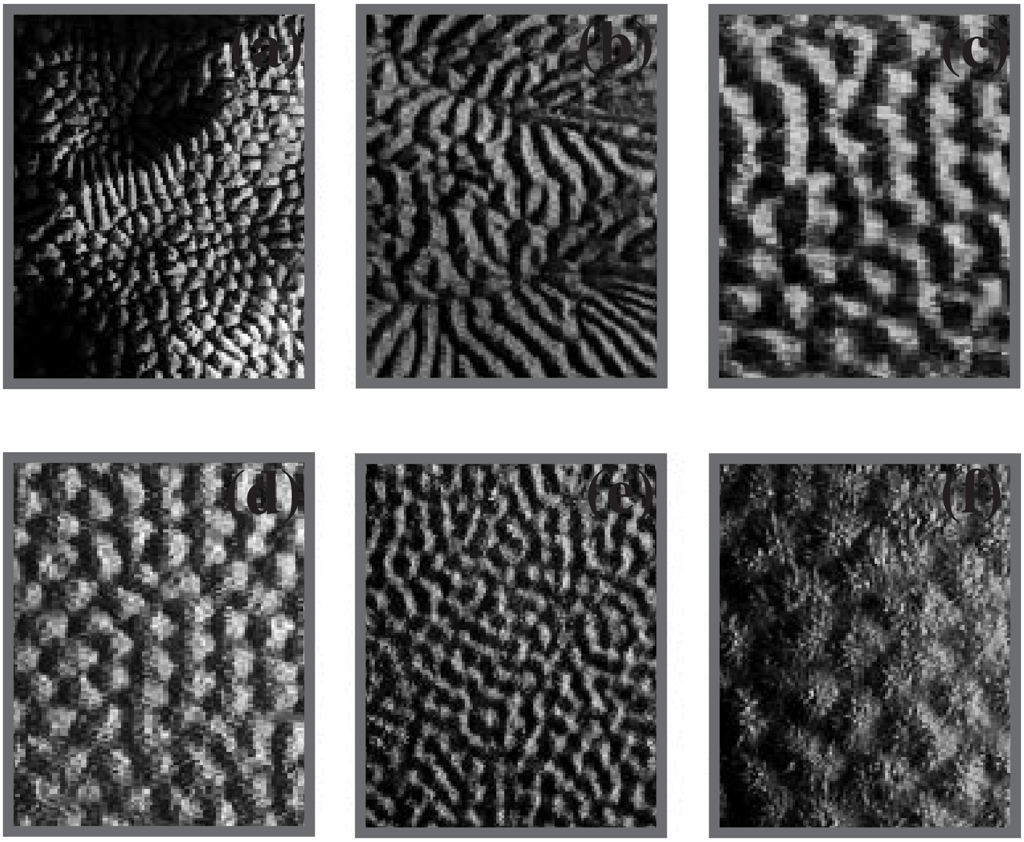

Fig. 4a–f. Snapshots of several states indicated in Fig. 3. States shown are referred in the text as **(a)** oozing cells, **(b)** stripes, **(c)** aligned cells, **(d)** staggered cells, **(e)** labyrinths and **(f)** cannibalizing cells. Details and scales of these photographs are the same as in Fig. 2

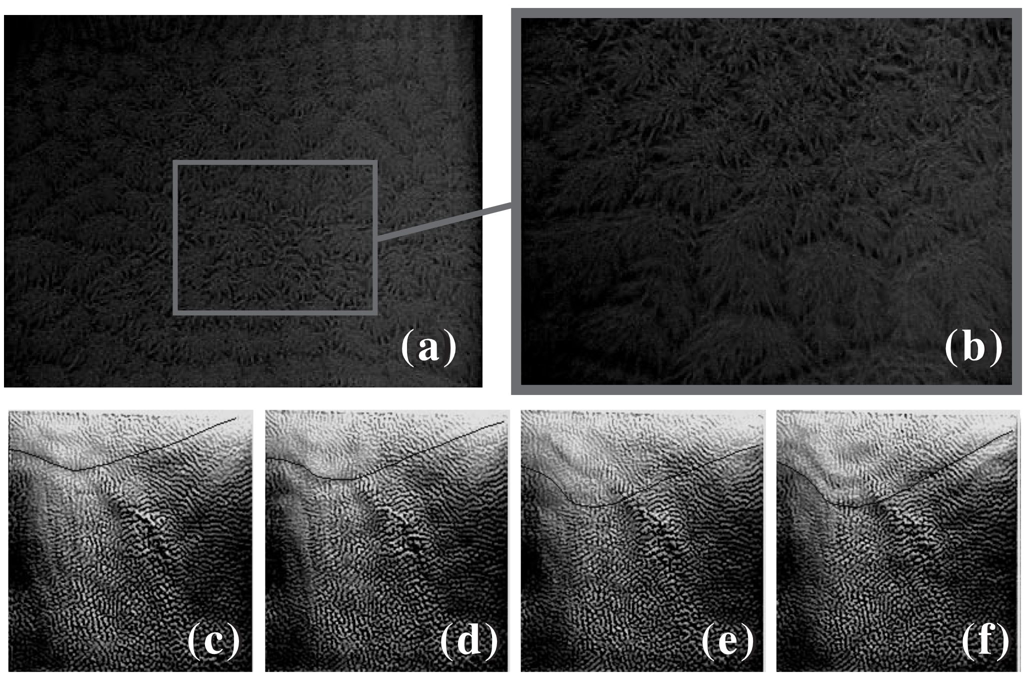

Fig. 5a–f. Details of cannibalizing and traveling patterns. **(a)** Overview and **(b)** detail of cannibalizing cells, showing network of fine filaments connecting individual cells. These snapshots were taken 1/2 driving cycle out of phase with Fig. 3(b) which shows the more coherent face of cannibalizing cells. **(c)**–**(f)** Time sequence of traveling wave state; snapshots are taken 2/15 sec. apart. The peaks of one such wave are identified by black curves

with harmonic driving does seem consistent with observations.

A final frequency subrange described in Fig. 3 is found between about 80 Hz and 100 Hz. Between about $\Gamma = 5$ and $\Gamma = 8$, we find traveling waves. We cannot account for their presence in the strongly damped granular bed, but they are easily observed, in two different guises. At frequencies close to 100 Hz, they are seen in an almost entirely flat and featureless bed, and at frequencies close to 90 Hz (shown in Fig. 5(c)–(f)) they appear superimposed on a finely crenulated surface. The vertical amplitude of these waves is under a millimeter, so to identify them in Fig. 5, we overlay a black curve on the peak of one of the waves [http://sol.rutgers.edu/∼shinbrot/DeepPatterns/Waves.mpg]. The waves emanate from the container sidewalls and rapidly and erratically shift direction as they flow; their wavelength is close to 1 cm, and their surface speed is of order 10 cm/sec. Also in this frequency range, we see labyrinthine patterns, as shown in Fig. 4(e).

To conclude, we have briefly surveyed the parameter space available in vibrated beds of fine grains. We have identified several new patterned states, however many features of these patterns remain to be explained. It is not at all clear how heaping, reported previously [2], and the new patterns inter-relate. Likewise is it not known how strongly dissipative grains manage to sustain oscillatory waves, or what provokes the transition between aligned and staggered cells. Among the new patterns reported in this letter, three seen in no other granular or fluid systems seem most illuminating.

First, there are the oozing cells shown in Fig. 4(a). Here we see manifestly that two very different effects co-exist: the heaping behavior that leads to large scale ridges out of which the grains ooze, and the interactions that produce smaller scale striped and cellular patterns – seen also independently of the oozing ridges. The mechanism by which multiple different effects co-exist in this system is not at all clear: on the one hand, this state lies at the borderline between gas- and grain-dominated flow, and it seems plausible that these flows produce heaps and cells respectively. At the highest frequencies explored, on the other hand, we again see quite distinct phenomena co-existing: the traveling waves shown in Fig. 5(c)–(f) are superimposed on a crenulated surface; identical waves appear at lower accelerations on an unfeatured, flat, surface. This reinforces the suggestion that surface dynamics (producing small scale patterns), subsurface dynamics (exhibited by traveling waves) and bulk dynamics (manifested by convection loops) may simultaneously be at work in generating the various patterns reported here. Yet at frequencies of 100 Hz, it is difficult to justify the argument that the dynamics are governed by air being pumped into and out of the bed.

More than this, the mere existence of these waves suggests two additional questions. One, granular materials are well known to be profoundly overdamped, thus it is remains to be seen how these waves sustain themselves. Two, in elastic or fluid media, there are identifiable sources of elastic stress; in the granular state it is not clear what provides the source of a restoring force that permits the waves to exist.

Finally, the cannibalizing cells shown in Fig. 4(f) occur at high accelerations as particles detach from the supporting bed and are not provided with sufficient time to thermalize before being energized again. As with the rapid flow shown in Fig. 1(b), this leads to energetic spraying of grains, producing the material transport between neighboring cells that leads to cannibalization. The presence of a transition from an equilibrated 'staggered cell' state in which particles remain in contact with the bed to this cannibalizing state in which particles detach from the bed is suggestive of a phase transition between a liquid-like granular state at lower accelerations and a gas-like state at higher accelerations [10]. Thus it seems worthwhile to hope that tools from a field that is well understood may in the future be brought to bear on the study of granular dynamics, which continues to present new surprises. Evidently much remains to be learned about the fundamental nature of granular dynamics.

References

1. P. B. Umbanhowar, F. Melo & H. L. Swinney: Localized excitations in a vertically vibrated granular layer. Nature, 382, 793 (1996)
   T. H. Metcalf, J. B. Knight & H. M. Jaeger: Standing wave patterns in shallow beds of vibrated granular material. Physica A, 236, 202 (1997)
   C. Bizon, M. D. Shattuck, J. B. Swift, W. D. McCormick & H. L. Swinney: Patterns in 3D vertically oscillated granular layers: simulation and experiment. Phys. Rev. Lett., 80, 57 (1998)
2. H. Pak, E. van Doorn & R. P. Behringer: Effects of Ambient Gases on Granular Materials under Vertical Vibration. Phys. Rev. Lett., 74, 4643 (1995)
   E. van Doorn & R. P. Behringer: Onset and evolution of a wavy instability in shaken sand. Phys. Lett. A P, 235, 469 (1997)
3. T. Shinbrot: Granular Coarsening. Granular Matter, 1, 145 (1998)
4. F. Dinkelacker, A. Hübler & E. Lüscher: Pattern formation of powder on a vibrating disc. Biol. Cybern., 56, 51 (1987)
5. J. B. Knight, E. E. Ehrichs, V. Y. Kuperman, J. K. Flint, H. K. Jaeger & S. R. Nagel: Experimental study of granular convection. Phys. Rev. E, 5726 (1996)
6. H. K. Pak & R. P. Behringer: Surface waves in vertically vibrated granular materials. Phys. Rev. Lett., 71, 1832 (1993)
7. R. M. Lueptow, A. Akonur & T. Shinbrot: PIV for Granular Flows, in press. Experiments in Fluids
8. D. Brone & F. J. Muzzio: Size Segregation in Vibrated Granular Systems: A Reversible Process. Phys. Rev. E, 56, 1059 (1997)
   T. Shinbrot & F. J. Muzzio: Reverse Buoyancy in Shaken Granular Beds. Phys. Rev. Lett., 81, 4365 (1998)
9. E. Falcon, K. Kumar, K. M. S. Bajaj & J. K. Bhattacharjee: Heap corrugation and hexagon formation of powder under vertical vibrations. Phys. Rev. E, 59, 5716 (1999)
   S. E. Esipov & T. Pöschel: The Granular Phase Diagram. J. Stat. Phys., 86, 1385 (1997)
10. H. M. Jaeger, S. R. Nagel & R. P. Behringer: Granular solids, liquids, and gases. Rev. Mod. Phys., 68, 1259 (1996)