ULTRAFAST LASER PULSE TRAIN RADIATION TRANSFER IN A SCATTERING-ABSORBING 3D MEDIUM WITH AN INHOMOGENEITY

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In this study, we continue investigating the ultrafast laser pulse train irradiation of inhomogeneous media, focusing on different degrees of inhomogeneity in a base scattering-absorbing medium. We used the transient discrete-ordinates method to acquire basic solutions of radiation transfer in the 3D inhomogeneous media subjected to a unit step pulse and then adopted the Duhamel’s superposition theorem to construct series solutions subjected to a pulse of different pulse widths or a pulse train consisting of many such pulses. The effects of scattering albedo, inhomogeneity size and location, pulse width and interval, and detector position on the temporal reflectance and transmittance signals are characterized and revealed.

KEY WORDS: discrete-ordinates method, superposition, short-pulse train, radiative transfer, inhomogeneity, heterogeneous medium

1. INTRODUCTION

The advent of ultrafast lasers has opened the window for many emerging applications. Specific applications in biomedicine include optical tomography (Yamada, 1995), tissue ablation (Sajjadi et al., 2013), tumor diagnostics (Bhowmik et al., 2014), thermal treatment (Jiao and Guo, 2009), to name a few. Recently, an excellent review on advances in the computational modeling of ultrafast radiative transfer was provided by Guo and Hunter (2013), in which advantages and disadvantages of various numerical solution methodologies have been summarized. The importance of studying the interaction of ultrashort-pulsed lasers with biological tissues and the corresponding mathematical modeling of possible thermal damage was also reviewed by Sajjadi et al. (2013). Some prior numerical investigations relevant to the present study are reviewed as follows.
In early studies, many researchers adopted the simplified diffusion approximation to model light transport in turbid tissues (Yamada, 1995). The diffusion approximation cannot describe the physical condition at the boundary such as light reflection and refraction, neither it is accurate when absorption is strong inside the medium (Guo et al., 2003). The transport of short light pulses through one-dimensional (1D) scattering-absorbing media using different approximate mathematical models, including the discrete-ordinates method (DOM), was examined by Mitra and Kumar (1999). The transient DOM for ultrashort-pulsed laser radiation transfer in 2D anisotropically scattering, absorbing, and emitting media with properties similar to those of the biological tissue was first formulated by Guo and Kumar (2001). The same authors later extended their transient DOM method to the 3D geometry (Guo and Kumar, 2002). The Fresnel effect and the heterogeneous effect under specularly and/or diffusely reflecting boundary conditions were further incorporated by Guo and Kim (2003).

The transport of a pulse train through a 2D rectangular participating medium consisting of a local inhomogeneity, by means of the finite-volume method (FVM) was numerically investigated by Muthukumaran and Mishra (2008). A complete thermal analysis combining the pulse train ultrafast radiative transfer and transient Pennes’ bioheat transfer in a model with a small tumor embedded in the skin tissue, using the transient DOM with an \( S_{10} \) scheme for radiation transfer and the FVM for bioheat transfer was developed by Jiao and Guo (2009). Recently, the transient heating on superficial tumors using the Monte Carlo method for solving the 3D radiative transfer problem was also investigated by Randrianalisoa et al. (2014). A numerical investigation of the thermal response of the biological tissue phantoms during laser-based pho-
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To-thermal therapy for destroying cancerous/abnormal cells was conducted by Kumar and Srivastava (2014). The presence of a tumor inside a breast tissue and its characteristics (e.g., location, size, and properties) with the skin surface temperature were estimated by Das and Mishra (2013). The temporal variations of transmittance and reflectance from a normal skin and a malignant human skin subjected to a low-power short-pulse laser using the modified DOM were analyzed by Bhowmik et al. (2014).

Duhamel’s superposition theorem was first introduced into the analysis of radiation transfer by Guo and Kumar (2002) to analyze the pulse feature of ultrafast radiative transfer, in which a basic solution to pulse irradiation was first estimated and then the superposition law was used to construct the responses of various pulses. Later, the DOM and Duhamel’s superposition theorem were applied by Akamatsu and Guo (2011) to scrutinize the transient characteristics of ultrafast radiative heat transfer in a homogeneous participating medium subjected to a diffuse short square pulse train. The transient characteristics of ultrafast laser radiative transfer in a homogeneous participating medium subjected to collimated irradiation of various square pulse trains by the DOM and Duhamel’s superposition theorem were clarified by Akamatsu and Guo (2013b). Ultrafast radiative transfer characteristics in multilayer inhomogeneous media subjected to a collimated short square pulse train by the DOM and Duhamel’s superposition theorem were reported by Akamatsu and Guo (2014). Akamatsu and Guo (2013a) also elucidated that Duhamel’s superposition theorem provides better accuracy and computational efficiency than direct pulse simulation for the radiative transfer analysis of multiple pulses.

The investigation and elucidation of ultrafast radiative transfer characteristics in a medium with an embedded inhomogeneity subjected to a pulse train are very important and practical to advance the applications of ultrafast laser technology in the biomedical field. Examples include laser cancer diagnosis, photodynamic therapy, laser hair removal, laser-induced interstitial thermotherapy, etc. To this end, we carried out a complete transient 3D numerical computations in the present study to understand the pulse train radiative transfer characteristics in scattering-absorbing media with an embedded inhomogeneity. The embedded medium may also be assumed to have the same optical properties like the surrounding medium to form a homogeneous medium for comparison purposes. Different pulse forms are considered, including a single collimated unit step pulse, a single collimated square pulse, and a train of collimated square pulses. We solved the problem by combining the DOM and Duhamel’s superposition theorem. The effects of the pulse width, pulse interval, scattering albedo, detecting position, and inhomogeneity size and location on the reflected and transmitted temporal signals were analyzed.

2. TISSUE MODELS CONSIDERED

The 3D geometry of the models considered in the present numerical computations is shown in Fig. 1a. The side length of the cubic medium is L, which is assumed to be
10 mm. The scattering albedo ($\omega = \sigma_s/\sigma_e$) of the base medium is set at either $\omega_1 = 0.9$ (highly scattering) or $\omega_1 = 0.1$ (highly absorbing). An inhomogeneity with scattering albedo $\omega_2$ varying from 0.1 to 0.9 is embedded inside the base medium. When $\omega_1$ is equal to $\omega_2$, the model tissue is homogeneous, i.e., no presence of inhomogeneity. When $\omega_1$ differs from $\omega_2$, the model tissue is embedded with an heterogeneous medium (such as a tumor). The extinction coefficient $\sigma_e ( = \sigma_a + \sigma_s)$ of both the base and embedded media is the same, equal to 1 mm$^{-1}$. The tissue scattering is reduced to isotropic (Guo and Kumar, 2000), since an effective scattering coefficient is used.

In the case of $\omega_2 = 0.1$ with $\omega_1 = 0.9$, the embedded medium has greater absorption than the base medium. On the other hand, when $\omega_2 = 0.9$ with $\omega_1 = 0.1$, the embedded medium has greater scattering than the surrounding medium. In general,
the biological tissue is highly scattering and very weakly absorbing against near-infra-red light, but a heterogeneous medium such as that of a malignant tumor is relatively highly absorbing and low-scattering. In the present numerical computations, we neglected the mismatch of refractive indices between two different media and both the media are assumed to be cold (i.e., medium emission is negligible).

The collimated laser square sheet with a width of \(d_c\), and \(d_c = L/49\) is normally incident at the central spot in the plane at \(x = 0\). Three pulse forms are considered: a single collimated unit step pulse, a single collimated square pulse, and a pulse train consisting of five square pulses with pulse width \(t_p\) and time interval \(t_d\) between two successive pulses. Three different inhomogeneity scenarios are also considered. In Fig. 1b, a small inhomogeneity with a side length of \(L/7\) is embedded at the center of the cubic medium. The reflected signals are detected at locations 2–7 on the incident wall of \(x = 0\), and the transmitted signals are detected at locations 1–7 on the opposite wall of \(x = L\). To examine the effects of the location and size of the embedded inhomogeneity, Fig. 1c considers an inhomogeneity of the same size like in Fig. 1b, but placed off the center, while Fig. 1d considers a larger inhomogeneity of side length \(21L/49\) placed at the cube center.

The governing equations and numerical schemes were well documented in our prior publications (Akamatsu and Guo, 2011, 2014) and thus, the details are not repeated here. The present method was previously verified by comparison with existing published results including the Monte Carlo simulation in a variety of exemplary problems in 2- and 3D systems (Guo and Kumar, 2001, 2002). The present study continues our previous study on inhomogeneous media (Akamatsu and Guo, 2014), with focusing (and difference) on the inclusion of an inhomogeneity to mimic the situation of a tumor surrounded by normal tissues. It is of practical importance in cancer optical diagnostics and imaging, and in photodynamic therapy.

3. RESULTS AND DISCUSSION

Figure 2 shows the temporal profiles of the reflectance detected by detectors 2–7 on the incident surface at \(x = 0\) of various situations when the small inhomogeneity is placed in the cube center (Fig. 1b). The incident laser pulse was a single collimated unit step pulse. The results were computed by the transient DOM with an \(S_{12}\) scheme. Because of the location symmetry, the signals in detectors 2–5 are identical. Signals at detectors 6 and 7 are different, with the strongest signal being at detector 6 and weakest at detector 7. It is seen that the reflectance signals are not affected by the inhomogeneity scattering albedo, but are determined by the base medium scattering albedo. The signal strength for the highly scattering base medium (\(\omega_1 = 0.9\)) is one to two orders of magnitude greater than that for the highly absorbing base medium (\(\omega_1 = 0.1\)).

Correspondingly, Fig. 3 shows the reflectance results when the small inhomogeneity is placed off the central position (see Fig. 1c). At all detector positions for either \(\omega_1 = 0.1\) or \(\omega_1 = 0.9\), there is no visible difference between Figs. 2 and 3.
FIG. 2: Temporal profiles of the reflectance detected by detectors 2–7 when the small inhomogeneity is located at the cube center: a single collimated unit step pulse: a–c) $\omega_1 = 0.9$; d–f) $\omega_1 = 0.1$
FIG. 3: Temporal profiles of the reflectance detected by detectors 2–7 when the small inhomogeneity is located at the upper region: a single collimated unit step pulse: a–c) $\omega_1 = 0.9$; d–f) $\omega_1 = 0.1$
The corresponding reflectance responses for the case with a large inhomogeneity at the central region (Fig. 1d) are displayed in Fig. 4. Apparently the scattering albedo of the large inhomogeneity strongly affects the signal strength, which increases with increasing inhomogeneity scattering albedo. In a highly scattering base medium (\(\omega_1 = 0.9\)), for instance, the ratio of the steady-state reflectance value computed for \(\omega_2 = 0.9\) to that computed for \(\omega_2 = 0.1\) was 1.023 in Fig. 4a, 1.098 in Fig. 4b, and 1.172 in Fig. 4c. In a relatively highly absorbing base medium (\(\omega_1 = 0.1\)), the corresponding ratio values were 1.028 in Fig. 4d, 1.452 in Fig. 4e, and 2.698 in Fig. 4f.

Figure 5 shows the temporal profiles of the transmittance detected by detectors 1–7 on the exit surface at \(x = L\) for the small inhomogeneity shown in Fig. 1b. The computational conditions are same like in Fig. 2. The magnitude of the transmittance decreased with increase in the distance between the laser-incident point and the detector location. The signals at detectors 2–5 are identical, but differences exist for detectors 1, 6, and 7. For transmittance, even with inclusion of a small inhomogeneity, the inhomogeneity scattering albedo affects the signals. The ratio of the steady-state transmittance value computed for \(\omega_2 = 0.9\) to that computed for \(\omega_2 = 0.1\) was 1.014 in Fig. 5a, 1.280 in Fig. 5b, 1.266 in Fig. 5c, 1.237 in Fig. 5d, 1.000 in Fig. 5e, 1.847 in Fig. 5f, 4.003 in Fig. 5g, and 5.918 in Fig. 5h.

The corresponding transmittance responses for the small inhomogeneity placed in the upper region (Fig. 1c) are illustrated in Figs. 6 (\(\omega_1 = 0.9\)) and 7 (\(\omega_1 = 0.1\)). For the tissue model in Fig. 1c, symmetry exists only for detectors 3 and 5, and thus, only the signals from these two detectors are identical. In Fig. 6, the ratio of the steady state transmittance value computed for \(\omega_2 = 0.9\) to that computed for \(\omega_2 = 0.1\) was 1.006 in Fig. 6a, 1.131 in Fig. 6b, 1.162 in Fig. 6c, 1.101 in Fig. 6d, 1.066 in Fig. 6e, and 1.168 in Fig. 6f. In Fig. 7, the ratio was 1.000 in Fig. 7a, 1.020 in Fig. 7b, 1.117 in Fig. 7c, 1.120 in Fig. 7d, 1.036 in Fig. 7e, and 1.280 in Fig. 7f. From Figs. 2 to 7 it is seen that the reflectance and transmittance will reach steady-state values within several dimensionless times when the incident laser pulse is an instantaneous unit step pulse.

Next, the characteristics of a single collimated square pulse with various pulse widths and a pulse train consisting of several square pulses are investigated. Their responses can be reconstructed using Duhamel's superposition theorem based on the results given in Figs. 2–7 for a unit step pulse. Figure 8 shows the temporal profiles of the reflectance in the medium with a large inhomogeneity subjected to a single collimated square pulse or a pulse train of five such pulses. The results are reconstructed using the basic solution shown in Fig. 4a. The corresponding figures reconstructed using the basic solution shown in Fig. 4e are shown in Fig. 9. The pulse width values were \(t_p^* = 0.03, 0.3, \) and 3.0, respectively. The pulse train intervals were \(t_d^* = t_p^*, 10t_p^*, \) and 100\(t_p^*\), respectively. Figures 8a and 8e and Figs. 9a and 9e show the temporal profiles for a single collimated square pulse. The rest of the panels in Figs. 8 and 9 show the temporal profiles for the pulse train.
FIG. 4: Temporal profiles of the reflectance detected by detectors 2–7 when the large inhomogeneity is placed at the cube center: a single collimated unit step pulse: a–c) $\omega_0 = 0.9$; d–f) $\omega_1 = 0.1$. 
FIG. 5: Temporal profiles of the transmittance detected by detectors 1–7 on the surface at $x = L$ when the small inhomogeneity is placed at the cube center: a single collimated unit step pulse: a–d) $\omega_1 = 0.9$; e–h) $\omega_1 = 0.1$
FIG. 6: Temporal profiles of the transmittance detected by detectors 1–7 when the small inhomogeneity is placed at the upper region with $\omega_1 = 0.9$: a single collimated unit step pulse
FIG. 7: Temporal profiles of the transmittance detected by detectors 1–7 when the small inhomogeneity is placed at the upper region with $\omega_1 = 0.1$: a single collimated unit step pulse
As seen in Figs. 8b, 8c, 8f, 8g, and Figs. 9b, 9c, 9f, 9g, the temporal reflectance signals from the irradiation of a pulse train differ greatly from those of a single pulse, due to the overlap effect. Such an overlap effect of the pulse train depends on the magnitude of the pulse broadening in response to the single square pulse. Therefore, the overlap effect in the reflectance signals shown in Fig. 8 is greater than those shown in Fig. 9, since the media had a higher scattering property in Fig. 8 than in Fig. 9. When the pulse train interval increased, however, the temporal reflectance signals produced by the irradiation of the pulse train completely agreed with those produced by the irradiation of a single pulse.

The temporal transmittance signals shown in Figs. 5b and 5f were also reconstructed by Duhamel’s theorem to get the responses to a square pulse or a pulse train. The results are shown in Figs. 10 and 11, respectively. A similar trend was observed in the transmittance signals as in the reflectance. Namely, with increase in the pulse train interval, the overlap effect of the transmitted pulse train gradually vanished, and the signals with five peaks were detected. It is seen that, when the pulse train interval is long enough compared to the pulse broadening, there is little overlap between the responses of successive pulses, and the characteristics of a pulse train are similar to those of a single pulse.

When we compare the reflectance signals shown in Fig. 8 to the transmittance signals shown in Fig. 10 for the inhomogeneous media with a relatively high scattering characteristic, the overlap effect in the transmittance is stronger. In addition, the differences of the peak value between the medium with an embedded heterogeneous medium (figures a to d) and the homogeneous participating medium (figures e to h) in Figs. 10 and 11 are significant compared to those shown in Figs. 8 and 9. When the inhomogeneity with a relatively high absorbing property is embedded in the highly scattering medium, its peak value in the left column is smaller than that in the right column under the same pulse width and the same pulse train interval, as seen in Fig. 10. On the contrary, when the inhomogeneity with a relatively high scattering property is embedded in a highly absorbing medium, its peak value in the left column is larger than that in the right column under the same pulse width and the same pulse train interval, as seen in Fig. 11.

4. CONCLUSIONS

We carried out 3D numerical computations to examine the transient radiative transfer in scattering-absorbing media with an embedded heterogeneous medium exposed to a collimated short pulse and/or pulse train by combining the transient DOM radiative transfer computation and Duhamel’s superposition theorem. The following conclusions can be drawn through comparison studies.

1) The difference in the reflectance signals between a medium with an inhomogeneity and a homogeneous medium is not clearly visible when the embedded inhomogeneity
FIG. 8: Temporal profiles of the reflectance at detector 6 when the large inhomogeneity is placed at the cube center: a single collimated square pulse or a pulse train of 5 collimated square pulses with base solution shown in Fig. 4a.
FIG. 9: Temporal profiles of the reflectance at detectors 2–5 when the large inhomogeneity is placed at the cube center: a single collimated square pulse or a pulse train of 5 collimated square pulses with base solution shown in Fig. 4e.
FIG. 10: Temporal profiles of the transmittance at detector 6 when the small inhomogeneity is at placed the cube center: a single collimated square pulse or a pulse train of 5 collimated square pulses with base solution shown in Fig. 5b
FIG. 11: Temporal profiles of the transmittance at detector 6 when the small inhomogeneity is placed at the cube center: a single collimated square pulse or a pulse train of 5 collimated square pulses with base solution shown in Fig. 5f.
neity is small. The reflectance signals are not affected by the inhomogeneity scattering albedo, but are determined by the base medium scattering albedo. The signal strength for the highly scattering base medium ($\omega_1 = 0.9$) is by one to two orders of magnitude greater than that for the highly absorbing base medium ($\omega_1 = 0.1$). However, the difference in reflectance is obvious when the inhomogeneity is large. Apparently the scattering albedo of the large inhomogeneity affects strongly the signal strength, which increases with increasing inhomogeneity scattering albedo, and this effect is more significant in highly absorbing base medium.

2) The difference in the transmittance signals between the medium with an inhomogeneity and a homogeneous medium is large even when the inhomogeneity is relatively small. The inhomogeneity scattering albedo strongly affects the signals.

3) The temporal reflectance signals due to the irradiation of a pulse train differ greatly from those of a single pulse, due to the overlap effect. Such an overlap effect of the pulse train depends on the magnitude of the pulse broadening in response to the single pulse as well as the pulse interval. When the medium has a scattering property, the pulse broadening is larger, and so is the overlap effect. The overlap effect in the transmittance is stronger than that in the reflectance. With increasing the pulse train interval, however, the overlap effect weakens. The temporal reflectance and transmittance signals produced by the irradiation of the pulse train may completely agree with those produced by the irradiation of a single pulse when the pulse interval is big enough.

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REFERENCES


