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Normalization of Various Phase Functions for Radiative Heat Transfer Analysis in a Solar Absorber Tube

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Normalization of various phase functions is considered for accurately predicting radiative heat transfer. A solar absorber tube filled with anisotropic scattering working medium is used as an example. Analysis of a previous normalization technique shows that while it does conserve scattered energy exactly after discrete-ordinates method (DOM) discretization, the overall asymmetry factor of the phase function is distorted, leading to substantial changes in overall scattering effect. A new normalization technique that conserves asymmetry factor and scattered energy simultaneously is investigated. The impact of lack of asymmetry factor conservation is analyzed for both the Legendre polynomial and the Henyey–Greenstein phase function approximations. Variations of medium optical thickness, scattering albedo, asymmetry factor, and side-wall emissivity are scrutinized to determine the effects of said parameters on wall heat flux and energy absorbing rate inside the absorber tube. Side-wall heat flux is found to increase with increases in asymmetry factor, optical thickness, and wall emissivity, and with decreases in scattering albedo. Energy absorbing rate profiles are found to depend greatly on optical thickness and scattering albedo.

INTRODUCTION

For many applications involving heat transfer, the contributions of radiation dominate those of conduction and convection. These processes range from determining the thermal efficiency and performance of solar collectors, combustion chambers, and crystal growth furnaces to the modeling the interaction of laser light with biological tissues [1–5]. When radiation is the dominant mode of heat transfer, accurate and complete solutions of the equation of radiative transfer (ERT) are necessary. Many methods have been introduced to solve the integro-differential ERT.

The discrete-ordinates method (DOM) was first introduced as an efficient tool in astrophysics [6] and neutron transport [7], and was later pioneered as a solution technique for radiative heat transfer mainly after the work of Fiveland [8]. A DOM study by Truelove [9] calculated radiative transfer in a three-dimensional enclosure. Use of the DOM for applications involving cylindrical enclosures was presented by Jamaluddin and Smith [10] and Jendoubi et al. [11]. Guo and co-authors [12–14] extended the use of the DOM to the solution of the time-dependent ERT for use in modeling ultrafast laser applications. Recently, Hunter and Guo [15] compared steady-state and ultrafast radiative flux profiles generated with the DOM to those by the finite-volume method to determine the efficiency and accuracy of each method.

Radiation scattering in most practical participating media is anisotropic. Special care must be taken to ensure that scattered energy is exactly conserved after the angular discretization of ERT. A common technique to guarantee conservation of scattered energy is phase function normalization. Previous publications have introduced normalization techniques that ensure scattered energy conservation [16, 17]. However, Boulet et al. [18] showed that these normalization procedures alter the overall asymmetry factor of the phase function, leading to distinct errors in calculated intensities and fluxes. To remedy this issue, Hunter and Guo [19, 20] introduced a new normalization technique that guarantees accurate conservation of scattered energy and phase function asymmetry factor.
The use of solar energy in place of traditional fossil fuels has become a highly researched area, due to concerns over pollution and global warming. In solar reactors and power plants, solar radiation is absorbed by solar absorber tubes via the use of reflectors and concentrators [21, 22]. The absorbed energy can be used to heat a working fluid such as molten salt or thermal oil. Jiang et al. [23] investigated the use of beam-splitting technology in a concentrated photovoltaic system to reduce the solar heating load on an absorber tube while increasing system efficiency. The work by Karni et al. [24] introduced a novel technology in a concentrated photovoltaic system to reduce the flow rate factor on the overall scattering effect, radiative heat flux, and energy absorbing rate inside the solar absorber tube is discussed. Radiative heat fluxes and energy absorption rates inside a simplified solar absorber tube. An analysis of the conservation of scattered energy and asymmetry factor is presented for in a concentrated solar absorber tube. An analysis of the conservation of scattered energy and asymmetry factor is presented for both the Legendre polynomial and the Heney–Greenstein (HG) phase function approximations with the different normalization techniques. The impact of a lack of conservation of asymmetry factor on the overall scattering effect, radiative heat flux, and energy absorbing rate inside the solar absorber tube is discussed. Radiative heat fluxes and energy absorption rates inside the solar absorber tube are determined for various asymmetry factors, side-wall emissivities, and medium optical thicknesses and scattering albedos.

**MATHEMATICAL FORMULATION**

Using the DOM, the steady-state ERT for a gray, absorbing–emitting and anisotropically scattering medium for any discrete direction \(\hat{s}^l\) can be written as [1]

\[
\frac{\mu}{r} \frac{\partial}{\partial r} [r I^l] - \frac{1}{r} \frac{\partial}{\partial \phi} [\eta^l I^l] + \xi \frac{\partial I^l}{\partial z} = \beta \left[ (1 - \omega) I_b - I^l + \frac{\omega}{4\pi} \sum_{i=1}^{M} w^i \Phi^{il} I^l \right]
\]

(1)

where the terms on the left-hand side represent the spatial gradient of radiative intensity; the first term on the right-hand side accounts for radiative energy contribution due to blackbody emission, the second term represents attenuation from both absorption and scattering, and the third term accounts for radiative energy in-scattering from direction \(\hat{s}^l\) into \(\hat{s}^l\); \(\mu^l, \eta^l, \) and \(\xi^l\) are the direction cosines corresponding to the \(r, \phi, \) and \(z\) directions, respectively; and \(\omega\) is the single scattering albedo, defined as the ratio of scattering coefficient \(\sigma_s\) to extinction coefficient \(\beta = \sigma_a + \sigma_s\). In the in-scattering sum, \(w^l\) is the DOM quadratic heat transfer engineering

\[
\Phi^{il} \approx \Phi_L(\Theta) = \sum_{i=0}^{N} C_i P_i(\cos \Theta)
\]

(2)

where \(\Theta\) is the scattering angle between directions \(\hat{s}^l\) and \(\hat{s}\), and the coefficients \(C_i\) are determined from Mie theory. Another widely used approximation is the HG phase function, which has been used to accurately capture the strong-forward scattering peak:

\[
\Phi^{il} \approx \Phi_{HG}(\Theta) = \frac{1 - g^2}{\left[1 + g^2 - 2g \cos \Theta\right]^{1.5}}
\]

(3)

where \(g\) is the phase function asymmetry factor, a measure of the average cosine of the scattering angle.

In order to ensure scattered energy conservation after DOM discretization, the discrete scattering phase function \(\Phi^{il}\) must satisfy the following conservation condition, for any given direction:

\[
\frac{1}{4\pi} \sum_{l=1}^{M} \Phi^{il} w^l = 1
\]

(4)

The discrete scattering phase function \(\Phi^{il}\) can be expressed as follows for an axisymmetric cylindrical medium:

\[
\Phi^{il} = \frac{1}{2} \left[ \Phi(\Theta_1) + \Phi(\Theta_2) \right]
\]

(5)

where the cosines of the scattering angles \(\Theta_1\) and \(\Theta_2\) are calculated using the following expressions:

\[
\cos \Theta_1 = \mu^l \mu^r \eta^l \eta^r + \xi^l \xi^r, \quad \cos \Theta_2 = \mu^l \mu^r - \eta^l \eta^r + \xi^l \xi^r
\]

(6)

The necessity of using two scattering angles comes from the axisymmetric nature of the problem, in which the discrete ordinate \(\eta\) is determined from the discrete ordinates \(\mu\) and \(\xi\) using the relationship \(\eta = \pm \sqrt{1 - \mu^2 - \xi^2}\). The square-root operator leads to two possible signs for each discrete ordinate \(\eta\), leading to the sign difference between the expressions for each scattering angle in Eq. (6).

In addition to satisfying the scattered energy conservation relation of Eq. (4), the discrete scattering phase function should also satisfy the following constraint to ensure that the phase function asymmetry factor \(g\) is conserved after DOM discretization:

\[
\frac{1}{4\pi} \sum_{l=1}^{M} \Phi^{il} \cos (\Theta^l) w^l = g
\]

(7)
Taking into consideration the two possible signs of $\eta$ once more, the product $\Phi^{l} \cos (\Theta^{l})$ can be expressed using an average of phase functions calculated at each individual scattering angle:

$$\Phi^{l} \cos (\Theta^{l}) = \frac{1}{2} [\Phi (\Theta_{1}) \cos \Theta_{1} + \Phi (\Theta_{2}) \cos \Theta_{2}] \quad (8)$$

Both the scattered energy conservation constraint of Eq. (4) and the asymmetry factor conservation constraint of Eq. (7) should be explicitly satisfied after DOM discretization in order to ensure that the solution of the ERT is accurate [19, 20]. Further details on the discretization and solution of the ERT using the DOM are not presented here, for brevity, but can be referenced from textbooks [1, 2] and journal publications [12–15].

**NORMALIZATION OF PHASE FUNCTION**

Figures 1a and 1b show seven Legendre polynomial phase function approximations that are to be considered in this study. The expansion coefficients for the three functions in Figure 1a were presented by Kim and Lee [25], while the expansion coefficients for the four phase functions in Figure 1b were given by Lee and Buckius [26]. The seven Legendre polynomial phase functions are defined by their asymmetry factors and the number of terms used in the polynomial expansion, as seen in Figures 1a and 1b. For comparison purposes, HG phase functions with the same asymmetry factors as the seven functions presented are also analyzed in this study to gauge the effectiveness of using each approximation.

As previously mentioned, as scattering becomes highly anisotropic, the scattered energy and asymmetry factor conservation constraints are not accurately satisfied. Table 1 shows the maximum directional deviation from the scattered energy conservation condition of Eq. (4) after DOM discretization using both the Legendre polynomial and HG phase function approximations. The values were obtained using the DOM S16 (288 total discrete ordinates) quadrature. For scattered energy to be exactly conserved, the values of the summation in Eq. (4) should be unity for all discrete directions. For $0 \leq \eta \leq 0.4$, scattered energy is effectively conserved for all directions when both the

<table>
<thead>
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<th>g (Prescribed)</th>
<th>Legendre</th>
<th>HG</th>
</tr>
</thead>
<tbody>
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<td>$N$</td>
<td>(%)</td>
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<td>0.000</td>
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</tr>
<tr>
<td>0.84534</td>
<td>13</td>
<td>2.700</td>
</tr>
<tr>
<td>0.927323</td>
<td>27</td>
<td>10.60</td>
</tr>
</tbody>
</table>

Figure 1 Legendre polynomial phase function approximations. (Color figure available online.)

Legendre polynomial and HG phase function approximations are implemented. However, as the asymmetry factor increases, deviations from unity appear. For the HG phase function, an increase in asymmetry factor produces a strict increase in maximum deviation from scattered energy conservation for all values of asymmetry factor, with a maximum deviation of 182.5% witnessed for $g = 0.927323$.

A similar linear correlation with asymmetry factor is not witnessed for the Legendre polynomial approximation. While deviations do appear for highly anisotropic scattering phase functions, there is also a dependence on the number of terms
in the Legendre expansion. The three functions in Figure 1a produce much smaller deviations from unity than the four functions in Figure 1b. For example, for the \( N = 9 \) expansion \((g = 0.669723)\), scattered energy is almost exactly conserved, with a maximum deviation of only 0.10%. Conversely, for the \( N = 34 \) expansion \((g = 0.4856)\), the maximum deviation reaches 2.80%, which differs from the pattern witnessed with the HG approximation. This indicates that both the asymmetry factor and the number of Legendre polynomial expansion terms (degree of oscillation in the actual shape) have an impact on the conservation of scattered energy after DOM discretization.

In order to ensure the accurate conservation of scattered energy, the scattering phase function must be normalized. Previous publications have introduced normalization techniques that guarantee the satisfaction of Eq. (4). The most widely used and best known simple technique, introduced by Kim and Lee [16], is to normalize the scattering phase function in the following manner:

\[
\tilde{\Phi}^{f l} = \Phi^{f l} \left( \frac{1}{4\pi} \sum_{l=1}^{M} \Phi^{f l} w' \right)^{-1}
\]  

(9)

A second technique, pioneered by Wiscombe [17], introduces corrective factors for each individual direction to normalize the scattering phase function. However, studies by Boulet et al. [18] and Hunter and Guo [19] showed that these techniques do not simultaneously ensure the conservation of asymmetry factor; that is, Eq. (7) is not accurately conserved after normalization. The deviation in overall asymmetry factor after discretization can lead to large errors in radiation analyses.

To remedy this issue, Hunter and Guo [19] introduced a new phase function normalization technique, which guarantees accurate conservation of both scattered energy and asymmetry factor simultaneously. The phase function is normalized in the following way:

\[
\tilde{\Phi}^{f l} = (1 + A^{f l})\Phi^{f l}
\]  

(10)

where the normalization parameter \( A^{f l} \) corresponds to scattering between two discrete directions. The normalized scattering phase function \( \tilde{\Phi}^{f l} \) is subject to the following constraints:

\[
\frac{1}{4\pi} \sum_{l=1}^{M} \tilde{\Phi}^{f l} w' = 1, \quad l' = 1, 2, \ldots, M
\]  

(11a)

\[
\frac{1}{4\pi} \sum_{l=1}^{M} \tilde{\Phi}^{f l} \cos (\theta^{f l}) w' = g, \quad l' = 1, 2, \ldots, M
\]  

(11b)

\[
\tilde{\Phi}^{f l} = \tilde{\Phi}^{f l}
\]  

(11c)

The first two constraints are identical to Eqs. (4) and (7), and the third constraint is a symmetry condition. The system of Eqs. (11a)–(11c) has more unknowns \((M^2 + M)/2\) than equations \((2M)\), and thus has infinitely many possible solutions. The desired normalization parameters that accurately satisfy conservation of scattered energy and phase function asymmetry factor can be determined by calculating the minimum-norm solution of the system of Eqs. (11a)–(11c) using either QR decomposition or the pseudo-inverse technique.

The importance of conservation of asymmetry factor can be seen in Table 2. According to the isotropic scaling law [27, 28], the reduced scattering coefficient is expressed as \((1 - g)\sigma_t\), and the change in overall scattering effect due to the lack of conservation in asymmetry factor can be determined by comparing values of \((1 - g)\) before and after DOM discretization. Table 2 shows both the overall asymmetry factor and change in scattering effect after discretization when normalization is ignored, as well as when both the old technique of Eq. (9) and Hunter and Guo’s new technique are implemented for both HG and Legendre phase functions.

For \( g = 0.4 \), asymmetry factor is conserved after discretization for both types of phase function approximations without normalization. As the asymmetry factor increases, discrepancies in the discretized asymmetry factor start to appear. For the HG phase function, the percent difference in scattering effect when normalization is ignored increases drastically as asymmetry factor increases, ranging from 0.102% at \( g = 0.4856 \) to 1054% at \( g = 0.927323 \). For the Legendre approximation, an increase in asymmetry factor from \( g = 0.4 \) produces an increase in change in scattering effect. However, as was the case for the scattered energy condition, the phase functions in Figure 1a show a smaller change in scattering effect than do the phase functions in Figure 1b after discretization. For example, the change in scattering effect when normalization is ignored for the \( N = 13 \) phase function \((g = 0.84534)\) is 2.095%. However, for the \( N = 26 \) expansion (which has a lower asymmetry factor...
$g = 0.818923$), the change in scattering effect is 11.92%. These results further the notion that the number of terms and physical shape of the Legendre approximation, as well as the asymmetry factor, have an effect on the conservation of both scattered energy and asymmetry factor after DOM discretization.

When the old normalization technique is implemented, the differences in scattering effect are drastically reduced, but are still noticeable, especially for high asymmetry factors. For the HG approximation, the percent changes in scattering effect are 2.362%, 4.953%, and 41.66% for $g = 0.818923$, 0.84534, and 0.927323, respectively. For the Legendre approximation, the percent changes in scattering effect are 2.472%, 0.284%, and 1.895% for the same prescribed asymmetry factors. These discrepancies, although much smaller than without phase function normalization, can still produce significant deviations in the radiative transfer results. When the new normalization technique is implemented, asymmetry factor is accurately conserved, leading to no change in scattering effect for either approximation.

Figures 2a and 2b plot the discretized phase function values calculated with both old and new normalization techniques for $g = 0.927323$. The theoretical phase function distribution is included for comparison purposes. The phase functions are plotted versus the cosine of $\Theta_1$ (the first scattering angle in Eq. (6)), and the use of the two scattering angles described earlier leads to the oscillatory behavior of the theoretical phase function. For the Legendre polynomial expansion in Figure 2a, both normalization procedures conform fairly accurately to the theoretical values, due to the fact that the overall asymmetry factor with the old normalization is $g = 0.9287$, which is only slightly different from the prescribed value. To clarify, the average percent difference in phase function value between the old normalization technique and the theoretical phase function value is 0.781%. When the new technique is implemented, this difference reduces to 0.321%.

When the HG phase function is considered in Figure 2b, an extreme discrepancy can be seen between the old and new normalization techniques. The old normalization produces a phase function profile that is drastically shifted from the theoretical, due to the fact that the asymmetry factor is altered from $g = 0.927323$ to $g = 0.9575$. These results for the HG phase function are consistent with the findings by Boulet et al. [18] and Hunter and Guo [19]. When the new normalization technique is considered, the discretized values match closely to the theoretical phase function. The preceding results mandate the necessity for ensuring the conservation of both scattered energy and asymmetry factor after DOM discretization, regardless of phase function approximation.

**RESULTS AND DISCUSSION**

The computing workstation used for the radiative transfer analysis is a Dell Optiplex 780, with an Intel 2 Dual Core 3.16 GHz processor and 4.0 GB of RAM. The DOM procedure was implemented using the FORTRAN computing language, and the values of the normalization parameters were determined by using MATLAB, and imported into FORTRAN. Since the normalization parameters depend solely on the quadrature scheme and phase function asymmetry factor, and not on the physical properties of the problem, they need only be determined once for a given phase function.

The test problem for this study involves the simple modeling of radiative transfer in an absorber tube in a solar energy power plant. The axisymmetric cylindrical tube contains a homogeneous participating medium. The tube has radius $R$ and length $L$,
The end walls of the tube are treated as mirrors, to account for the neglecting of end effect because a real absorber tube is quite long. The use of mirror end walls also means that heat flux profiles will be constant along the length of the tube, so changes in tube aspect ratio can be neglected. The spatial grid considered was \((N_r \times N_z) = 40 \times 40\) for all simulations. The solar radiant heating is approximated as a diffuse boundary condition on the side-wall of the enclosure, with surface temperature \(T_w\), typically in the range from 500 K up to 1200 K depending on the solar receiver condition. The heat flux is nondimensionalized by the wall emissive power and absorber tube radius, as

\[
q^* = -q_g / \sigma T_w^4
\]

The wall of the absorber tube is usually metallic and thin, so that heat loss due to conduction through the wall is negligible. The participating medium housed inside the absorber tube has optical thickness \(\tau(T = \beta R)\) and scattering albedo \(\omega\), and the inner surface of the side wall is taken to have emissivity \(\epsilon_w\).

These properties are later varied to gauge the effect of different material properties on the absorption of solar energy. Moreover, the medium is assumed to be cold throughout, to show solar radiative heat transfer only.

It is beneficial to examine the effect of the normalization techniques on radiative transfer results to further demonstrate the absolute necessity of conserving both scattered energy and asymmetry factor. Table 3 lists the percentage difference of side-wall heat flux between different combinations of phase function approximations and normalization techniques. When comparing the Legendre approximation with old and new normalization, we see that the percentage differences in heat flux are extremely small for both the optical thicknesses. The maximum differences occur for \(g = 0.818923\) in both cases, with values of 0.482% and 0.661% for \(\tau = 10.0\) and \(\tau = 25.0\), respectively. When comparing the old and new normalized HG phase function heat fluxes, a strict increase is again seen as asymmetry factor increases, with the differences reaching 2.652% and 4.848% for \(g = 0.927323\) for the two optical thicknesses. When the heat fluxes calculated with the HG and Legendre with new normalization are compared, we see that the percentage difference is less than 0.185% for all asymmetry factors and optical thicknesses, leading to the conclusion that the choice of phase function approximation has little effect on the wall heat flux for these given conditions. In general, the difference in side-wall heat flux when old and new normalizations are implemented is small, due to the magnitude of the heat flux stemming from the high emissive power of the wall.

A comparison of the radiant energy absorption rate inside the participating medium is also performed for both phase function approximations with either old or new normalization in Table 4, in which the maximum percentage difference of radiation absorption rate along the absorber tube radius is listed. This comparison is of particular importance for the solar absorber, as absorbed energy is directly transferred to the working fluid in order to power other processes in the solar power plant. The volumetric radiative energy absorption rate is nondimensionalized by the wall emissive power and absorber tube radius, as

\[
\delta_{rad}^* = \frac{\delta_{rad} R}{(4 \pi I_{bw})}
\]

The data in Table 4 indicate that normalization technique has a larger impact on the energy absorption rate than on the wall heat flux. When comparing the Legendre approximation with old and new normalization for \(\tau = 10.0\), the largest difference is 2.769% for \(g = 0.818923\). The previously seen pattern between the phase functions in Figures 1a and 1b is again witnessed. When the HG approximation is used, much larger differences in energy absorption rate occur for highly anisotropic scattering, with the percentage difference reaching a maximum of 9.718% at \(g = 0.927323\). These larger differences result from the lack of conservation of asymmetry factor, and mandate the necessity of accurate conservation. When comparing the HG to the Legendre with the new normalization technique, differences are less than 1% for all asymmetry factors except \(g = 0.485623\). The larger difference of 1.546% likely stems from the backward scattering lobe of the Legendre phase function, as the HG phase function for the same asymmetry factor does not contain this lobe. In general, however, the excellent correlation between results calculated with both the HG and Legendre approximations with new normalization indicates that the two approximations can be implemented interchangeably without distinct consequences for this optical thickness.

### Table 3

<table>
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<tr>
<th>(g)</th>
<th>(N)</th>
<th>(\tau = 10.0)</th>
<th>(\tau = 25.0)</th>
<th>(\tau = 10.0)</th>
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<td>0.221</td>
<td>0.352</td>
<td>2.652</td>
<td>4.848</td>
</tr>
</tbody>
</table>

### Table 4

| Maximum Percent Difference in divergence of heat flux (%) | \(|\frac{|\text{LogOld} - \text{LogNew}|}{\text{LogNew}}\) | \(|\frac{|\text{HGOld} - \text{HGNw}|}{\text{HGNw}}\) | \(|\frac{|\text{LegNew} - \text{HGNw}|}{\text{LegNew}}\) |
|--------------------------------------------------------|---------------------------------|---------------------------------|----------------|
| \(g = 0.4\)                                          | 0.4                             | 3.35                           | 5.54            |
| \(g = 0.669723\)                                      | 0.927323                        | 3.07                           | 4.75            |
| \(g = 0.818923\)                                      | 0.818923                        | 2.77                           | 4.22            |
| \(g = 0.927323\)                                      | 0.927323                        | 2.76                           | 4.75            |
The same conclusion, however, cannot be reached when the optical thickness increases to $\tau = 25.0$. The maximum difference between Legendre with old and new normalization again occurs for $g = 0.818923$, with a value of 9.680%. The maximum difference between HG with old and new normalization occurs for $g = 0.927323$ (as expected from the previous results), and has a value of 53.873%, indicating that a lack of conservation of asymmetry factor has a much larger impact when the medium is optically thick. The patterns noticed in percent difference between the various Legendre phase functions are identical to the previous analysis. The major difference, however, lies in the comparison between the Legendre and HG approximations with new normalization. For the thinner medium, it was witnessed that the differences in absorbed energy rate were minor for all cases except for $g = 0.485623$, where the major backward phase function lobe played a large role in the difference between the results calculated with the two approximations. For the thicker medium, however, the differences between the two approximations are much larger, reaching more than 6% for both $g = 0.485623$ and $g = 0.818923$. This is due to the fact that the thicker medium has the capability to absorb more energy, and that the radiant energy is not able to fully propagate through the medium, meaning that subtle changes in the phase function (such as the difference in overall shape between the HG and Legendre approximations) can lead to larger discrepancies in physical results. As optical thickness increases, we cannot conclude that each phase function approximation can be used interchangeably due to the discrepancies seen in the absorbed energy rate results.

As previously mentioned, there are various materials that can be added to the inside of a solar absorber tube to enhance the absorption of radiant energy. A parametric study was performed to gauge the impact of different material properties and side-wall emissivity on both the heat flux leaving the side wall and the volumetric radiative energy absorption rate along the absorber tube radius. For this parametric study, the Legendre polynomial approximation was implemented with the new normalization technique.

Figure 3 examines the change of nondimensional radiative energy absorption rate versus radial location for changes in asymmetry factor. The medium was taken to have optical thickness $\tau = 10.0$, scattering albedo $\omega = 0.92$, and the side-wall emissivity was taken to be $\varepsilon_w = 0.99$. Far away from the side wall, the amount of energy absorbed by the medium increases as asymmetry factor increases. This is due to the fact that as scattering becomes more highly anisotropic, the radiant energy is more strongly scattered away from the side wall, leading to higher intensity values near the radial centerline and lower intensity values near the side wall. This explains the opposite trend near the side wall, where the largest radiative energy absorption rate occurs for $g = 0.4$, and the smallest for $g = 0.927323$.

Figure 4 plots the variations of side-wall non-dimensional heat flux versus phase function asymmetry factor for varying medium optical thickness ($\tau = 1.0$ to 100). The scattering albedo and wall emissivity are kept fixed at $\omega = 0.92$ and $\varepsilon_w = 0.99$. For all asymmetry factors, the nondimensional heat flux emanating from the side wall increases in magnitude as the optical thickness increases, since more radiant energy can be captured by the medium. The increase in heat flux as optical thickness increases is more pronounced for larger asymmetry factors.

Figure 5 plots the nondimensional radiative energy absorption rate versus radial location for various optical thicknesses with $g = 0.927323$. Near the side wall, increases in optical thickness produce increases in energy absorption rate, similar to the results for heat flux seen in Figure 4. For optical thickness less than $\tau = 10.0$, radiant energy is able to propagate strongly throughout the entire medium, leading to flatter radial
energy absorption rate profiles. Conversely, for the extremely thick medium with $\tau = 100$, the energy absorption rate at the side wall is five orders of magnitude higher than at the radial centerline.

Figure 6 examines side-wall heat flux versus phase function asymmetry factor for varying values of medium scattering albedo. The scattering albedo was varied from $\omega = 0.98$, where scattering is dominant, to $\omega = 0.68$, where absorption has a significant effect. The optical thickness and wall emissivity are kept constant at $\tau = 10.0$ and $\varepsilon_w = 0.99$. As scattering albedo decreases, the magnitude of the heat flux emanating from the side wall increases due to the ability of the medium to absorb more radiant energy. In contrast to the results shown for variations in optical thickness, the increase in heat flux with change in scattering albedo is more pronounced for smaller asymmetry factors. Heat flux away from the wall again increases as asymmetry factor increases. For $\omega = 0.98$, where the effect of absorption is far outweighed by scattering, however, the effect of asymmetry factor starts to become less significant.

Variations of radiative energy absorption rate with changes in scattering albedo are examined in Figures 7a and 7b for...
optical thickness $\tau = 1.0$ and $\tau = 10.0$, respectively. For the thin medium in Figure 7a, we see that increases in scattering albedo produce decreases in energy absorption rate for all radial locations. For $\omega = 0.98$, the energy absorption rate is fairly similar across the entire radial distance. As albedo decreases, and absorption becomes more dominant, the energy absorption at the side wall increases when compared to that at the centerline. For the optically thicker medium in Figure 7b, the energy absorption rate is highly concentrated near the side wall, excluding the case where $\omega = 0.98$.

Figure 8 plots side-wall heat flux versus asymmetry factor for various side-wall emissivities. The optical thickness and scattering albedo of the medium are kept constant at $\tau = 10.0$ and $\omega = 0.92$. As wall emissivity increases, the heat flux emanating from the side wall also increases due to the direct dependence of the side-wall emissive power on the wall emissivity. Once again, as asymmetry factor increases, the heat flux is scattered more strongly away from the wall, leading to the witnessed increase in heat flux profile. The increase in heat flux with increasing wall emissivity becomes slightly more pronounced as asymmetry factor is increased.

Figure 9 plots the radiative energy absorption rate versus radial location for various asymmetry factors and wall emissivities. The energy absorption rate increases with increasing wall emissivity. The optical thickness of the medium limits the propagation of radiant energy, leading to the opposite trends for varying asymmetry factor at the centerline and the side wall. As asymmetry factor increases, the difference in energy absorption rate between $\varepsilon_w = 0.99$ and $\varepsilon_w = 0.80$ increases.

CONCLUSIONS

In this study, normalization of Legendre and HG phase functions was implemented in conjunction with the DOM to perform radiative heat transfer analysis inside a solar absorber tube. The following conclusions can be made:

1. For cases where scattering is highly anisotropic and oscillatory, it is essential that the phase function normalization procedure must conserve both scattered energy and asymmetry factor after directional discretization. When the new normalization technique of Hunter and Guo (2012) is implemented, both asymmetry factor and scattered energy are accurately conserved regardless of whether the phase function is approximated using the Legendre or HG format.
2. For the HG approximation, the importance of appropriate normalization increases as asymmetry factor increases. For the Legendre approximation, the number of terms (i.e., degree of shape oscillation) as well as the asymmetry factor have a distinct impact on conservation after normalization.
3. When calculating heat fluxes and absorbed energy rate profiles, the Legendre approximation and HG approximation produce similar results when optical thickness is not greater than 10.0. As optical thickness increases, the overall shape of the phase function becomes more significant, and the results calculated using the two different phase function approximations start to deviate.
4. Heat flux emanating from the side wall and overall radiation absorption increase with increasing medium optical thickness, wall emissivity, and asymmetry factor, and decrease for increasing scattering albedo.

NOMENCLATURE

$A^{fl}$ normalization coefficients
$g$ asymmetry factor
$I$ radiative intensity (W/m²-sr)
The image contains a page from a document discussing radiative heat transfer. The page includes mathematical symbols, descriptions of physical quantities, and references to various sources. Below is the transcription of the page's content:

**Greek Symbols**

- $\beta$: extinction coefficient, $= \sigma_a + \sigma_s$ (m$^{-1}$)
- $\varepsilon$: emissivity
- $\mu$, $\eta$, $\xi$: direction cosines
- $\Phi$: scattering phase function
- $\Phi^*$: normalized scattering phase function
- $\phi$: azimuthal angle (rad)
- $\tau$: optical thickness, $= \beta R$
- $\Theta$: scattering angle (rad)
- $\omega$: scattering albedo, $= \sigma_s / \beta$

**Subscripts**

- $b$: blackbody
- HG: Henyey–Greenstein
- L: Legendre
- $w$: boundary wall

**Superscripts**

- $i$: radiation incident direction
- $l, l'$: radiation directions
- $l''$: from direction $3l''$ into direction $3l$
- *: non-dimensional quality

**REFERENCES**


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