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Analysis of the Nusselt number in pulsating pipe flow

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1. INTRODUCTION

Much attention has been given to the possibility of enhancing the heat transfer rate by superimposing pulsation on a mean flow inside a confined passageway. Industrial applications can be found in Stirling engines and reciprocating cycles, to name a few. A perusal of the relevant literature reveals that pulsating flows in a pipe and the attendant heat transfer have been the subject of several analytical [1, 2] and experimental investigations [3, 4]. However, the available published data have been inconclusive and they often show conflicting results. Some investigators reported heat transfer enhancements [4, 5], whereas heat transfer reductions were also noted by some workers [6]. In some instances, both heat transfer augmentation and reduction were detected in a single experiment [3]. Some of these discrepancies can be traced to differences in the parameter spaces that were examined, as well as the nonuniformities in experimental methodologies utilized in various research efforts. However, an analysis of these discrepancies is of vital importance for a more complete application to pulsating heat transfer problems. The present Technical Note addresses this technical issue.

One promising avenue to cope with these discrepancies is to reevaluate the Nusselt numbers used in prior published works. It is found that many versions of the Nusselt number were devised to account for the results of the analyses. For a small-amplitude pulsating flow, the Nusselt numbers in various forms are shown to lead to inconsistent results. In addition, if the pulsation amplitude is appreciable, a reverse flow is produced. In this case, difficulties arise in defining the bulk temperature or the time-dependent Nusselt number over a cycle. The present Technical Note aims to examine the consequences of using various forms of the Nusselt number. Special attention is paid to the case of a large-amplitude pulsation flowrate ($A_f \geqslant 1$), which causes reverse flows at the cross section in a pipe. A new definition of the Nusselt number is proposed in the present study. By adopting this definition, the influence of the pulsation amplitude and frequency on heat transfer is scrutinized.

2. MATHEMATICAL FORMULATION AND NUMERICAL SCHEME

For an unsteady, laminar axisymmetric, fully developed pipe flow, the governing equations can be written in dimensionless form as

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{2}{Re}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \tag{1}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \frac{2}{Re\,Pr}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right). \tag{2}$$

In the above, the non-dimensional variables are defined as

$$x = \frac{x^*}{a} \quad r = \frac{r^*}{a} \quad u = \frac{u^*}{\bar{u}_s^*} \quad t = \frac{t^*}{a/\bar{u}_s^*} \quad p = \frac{p^*}{\rho \bar{u}_s^{*2}} \quad T = \frac{T^*}{q''a/k}$$

where the asterisk denotes the dimensional counterpart, and $\bar{u}_s^*$is the space-averaged steady-state velocity. The fluctuating pressure gradient is given as

$$-\frac{\partial p}{\partial x} = \left(\frac{dp}{dx}\right)_m \left[1 + A_0 \sin\left(\frac{2\beta^2}{Re}t\right)\right] \tag{3}$$

where A_0 represents the pulsation amplitude of pressure gradient, and β indicates the frequency parameter ($\beta = a\sqrt{\omega/\nu}$). Based on the pulsation due to pressure gradient in equation (3), the space-averaged velocity ($\bar{u}$) can be expressed as

$$\bar{u} = 1 + A_f \sin\left(\frac{2\beta^2}{Re}t + \phi\right)$$

where A_f is the pulsation amplitude, and ϕ is the phase of $\bar{u}$ with respect to the pressure pulsation. When $A_f \geqslant 1$, reverse flows are encountered at some times. It should, however, be noted that since the pulsating component is superimposed on a mean flow, the time-and space-averaged velocity is always positive.

The proper boundary conditions are as follows:

$$\frac{\partial u}{\partial r} = 0 \quad \frac{\partial T}{\partial r} = 0 \quad \text{at } r = 0$$

$$u = 0 \quad q'' = \text{constant} \quad \text{at } r = 1. \tag{4}$$

To solve the above system of equations, Chatwin's approximation [7] was employed, i.e.

$$\frac{\partial T}{\partial x} = \frac{4}{Re\,Pr} = \text{constant}. \tag{5}$$

The fully implicit scheme based on a control-volume formulation [8] was introduced to discretize the governing equations. The resulting algebraic equations were solved by the tridiagonal matrix algorithm. In the radial direction, nonuniform grid points were densely packed near the pipe wall. The number of mesh points was 100. To ensure sufficient

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NOMENCLATURE

a	pipe radius [m]
A_f	pulsating amplitude ratio of flowrate
A_0	pulsating amplitude ratio of pressure gradient
h	heat transfer coefficient
k	thermal conductivity of fluid [W m^{-1} K^{-1}]
Nu	Nusselt number, $2ah/k$
p	pressure
Pr	Prandtl number
q''	heat flux [W m^{-2}]
r,x	coordinates
Re	Reynolds number, $2a\bar{u}_s^*/\nu$
t	time
T	temperature
u	velocity
$\bar{u}_s^*$	mean value of steady velocity [m s^{-1}].

Greek symbols

β	Womersley number, $a\sqrt{\omega/\nu}$
ν	kinematic viscosity [m^2 s^{-1}]
ρ	fluid density [kg m^{-3}]
ϕ	phase angle [rad]
ω	angular velocity [rad s^{-1}]
Ω	pulsation cycle angle, ωt^*.

Subscripts

b	bulk
m	time-mean value
s	steady state
t	time-dependent
w	wall.

Superscript

*	dimensional quantity.

temporal resolution, 3600 time intervals were utilized to constitute one pulsating cycle for higher frequencies, whereas 360 time steps were used for lower frequencies. The temporally periodic solution was usually attained after 20 cycles of pulsation. For convergence criteria, the relative variations in velocity and temperature between two successive iterations were smaller than the pre-assigned accuracy levels of 10^{-4}. In actual calculations, a steady-state solution was adopted as the initial-state condition. Trial calculations were repeated to test the sensitivity of the results to grid size and time interval. The outcome of these was satisfactory. The numerical results were validated by checking them against the analytical solutions of Uchida [1]. The Reynolds number was set as $Re = 500$: $Pr = 7.0$ in the present computation.

3. THE NUSSELT NUMBER DEFINITIONS

3.1. *The first kind of Nusselt number* (Nu_1)
In the case of constant wall heat flux, the Nusselt number is generally defined as $Nu = 2/(T_{wm}-T_{bm})$, in which T_{wm} denotes the time-averaged temperature at the wall and T_{bm} is the time-averaged bulk temperature [5, 6]. These can be decomposed into $T_{wm} = T_{ws}+\Delta T_w$, and $T_{bm} = T_{bs}+\Delta T_b$. The difference ΔT_w is generated due to the interaction of pulsating velocity and pulsating temperature over a cycle ($\Omega = \omega t^*$), which can be written as

$$\Delta T_w = \frac{-\int_0^{2\pi}\int_0^1 (T-T_s)(u-u_s)r\,dr\,d\Omega}{\int_0^{2\pi}\int_0^1 u_s r\,dr\,d\Omega}. \quad (6)$$

The time-mean bulk temperature is defined as

$$T_{bm} = \frac{\int_0^{2\pi}\int_0^1 Tur\,d\,r\,d\,\Omega}{\int_0^{2\pi}\int_0^1 ur\,d\,r\,d\,\Omega}. \quad (7)$$

By employing the heat balance equation $\int_0^{2\pi}\int_0^1(Tu-T_s u_s)r\,d\,r\,d\,\Omega = 0$, the Nusselt number ($Nu_1$) can be obtained:

$$\Delta T_b = 0 \quad Nu_1 = \frac{2}{T_{ws}+\Delta T_w-T_{bs}}. \quad (8)$$

Siegel [6] and Kurzweg [7] showed that $\Delta T_w > 0$. It means that Nu_1 is always less than the Nusselt number for a steady state (Nu_s). In other words, heat transfer is reduced when pulsation is superimposed to a steady flow.

3.2. *The second kind of Nusselt number* (Nu_2)
Since the bulk temperature floats unsteadily over a pulsation cycle, it is more useful to define the time-dependent bulk temperature (T_{bt}) and the time-averaged one (T_{bm}) as [2]

$$T_{bt} = \frac{\int_0^1 uTr\,d\,r}{\int_0^1 ur\,d\,r} \quad T_{bm} = \frac{\int_0^{2\pi} T_{bt}\,d\,\Omega}{\int_0^{2\pi} d\Omega} \neq T_{bs}. \quad (9)$$

Hence, the Nusselt number (Nu_2) has the form.

$$Nu_2 = \frac{4\pi}{\int_0^{2\pi} T_{wt}\,d\Omega - \int_0^{2\pi} T_{bt}\,d\Omega} = \frac{2\pi}{\int_0^{2\pi}(1/Nu_t)\,d\Omega} \quad (10)$$

in which the time-dependent Nusselt number is $Nu_t = 2/(T_{wt}-T_{bt})$.

3.3. *The third kind of Nusselt number* (Nu_3)
Next, an alternate approach is pursued for the averaged time-dependent Nusselt number (Nu_3) over a pulsation cycle [2, 3]:

$$Nu_3 = \frac{\int_0^{2\pi} Nu_t\,d\Omega}{\int_0^{2\pi} d\Omega} = \frac{\int_0^{2\pi}\frac{2}{T_{wt}-T_{bt}}\,d\Omega}{2\pi}. \quad (11)$$

A closer comparison of equations (11) and (10) indicates that Nu_3 is an arithmetic mean value, whereas Nu_2 is a harmonic mean one. Obviously, $Nu_2 < Nu_3$, since all the time-dependent Nusselt numbers (Nu_t) cannot have the same value. When the flow is pulsated, discrepancies exist among Nu_1, Nu_2 and Nu_3. In particular, when a reverse flow is present ($A_f \geqslant 1$), difficulties arise in calculating T_{bt}, i.e. it goes to infinity at some times or is even equivalent to the time-dependent wall temperature (T_{wt}). This brings forth discontinuities in the profiles of T_{bt} or Nu_t during a cycle. As a result, Nu_2 and Nu_3 are not suitable for a large-amplitude pulsation.

3.4. The fourth kind of Nusselt number (Nu_4)

To overcome the above-mentioned drawback, a new definition of the time-dependent bulk temperature is proposed in the present study:

$$T_{bt1} = \frac{\int_0^1 T\sqrt{u^2}\,r\,dr}{\int_0^1 \sqrt{u^2}\,r\,dr}. \quad (12)$$

As can be seen in equation (12), the bulk temperature correctly reflects an integrated mass–energy average temperature. When A_f is small ($u > 0$ at any time), T_{bt1} conforms with T_{bt}. When equation (12) is employed, it is found that inconsistency of the profiles of T_{bt1} or Nu_t can be avoided for a large-amplitude of pulsation ($A_f \geqslant 1$). With this new definition, the arithmetic mean Nusselt number (Nu_4) is calculated over a cycle:

$$Nu_4 = \frac{\int_0^{2\pi} Nu_t\,d\Omega}{\int_0^{2\pi} d\Omega} = \frac{1}{\pi}\int_0^{2\pi} \frac{1}{T_{wt} - T_{bt1}}\,d\Omega. \quad (13)$$

4. DISCUSSION

The performance of the above-defined Nusselt numbers, by applying the same heat transfer condition to a pulsating pipe flow, is now inspected. Figure 1 depicts four Nusselt number distributions, where the steady-state Nusselt number (Nu_s) is subtracted from each Nusselt number. If the difference is positive (negative), it means that heat transfer is augmented (reduced). The pulsating amplitude is varied in a range of $0 \leqslant A_f \leqslant 1.0$ for three pulsating frequencies ($\beta = 1, 3$ and 6). As is evident in Fig. 1, the computed Nusselt numbers are not consistent. If Nu_1 and Nu_2 are employed, the heat transfer due to pulsation is generally reduced as A_f increases, except for the case of a low frequency ($\beta = 1$) for Nu_2. However, the effect of β on heat transfer between Nu_1 and Nu_2 is shown to be reversed compared to each other. For example, the effect of pulsation on heat transfer at high frequency ($\beta = 6$) is not appreciable for $Nu_1 - Nu_s$, but it is significantly reduced at $\beta = 6$ for $Nu_2 - Nu_s$. Comparing Fig. 1(b) with Fig. 1(c), it is seen that $Nu_2 < Nu_3$. For the cases of Nu_3 and Nu_4, both heat transfer enhancement and reduction are detected. It is seen that the profiles of $Nu_3 - Nu_s$ and $Nu_4 - Nu_s$ are nearly similar at small A_f ($A_f \leqslant 0.5$). However, different trends are displayed as A_f increases ($\beta = 3$ and 6). In summary, depending upon the choice of the definition of the Nusslet number, the overall results can be interpreted differently.

Next, as the pulsation amplitude is increased further ($A_f > 1$), the results illustrate dramatic changes. To validate the present Nusselt number (Nu_4), a comparison is made with the experiment of Kim [9] for $A_f = 1.05$ and 2.05. As shown in Table 1, Nu_4 gives a reasonable agreement with the experiment. As stated earlier, Nu_2 and Nu_3 are meaningless when $A_f \geqslant 1$. Moreover, Nu_1 is also reduced even for a large pulsation amplitude ($A_f \geqslant 1$).

Comparisons are extended to the profiles of $Nu_4 - Nu_s$ at a large amplitude ($0 \leqslant A_f \leqslant 2.0$). As seen in Fig. 2, Nu_4 is always larger than Nu_s in the high-amplitude range ($A_f \geqslant 1$), i.e. the heat transfer due to pulsation is always augmented. For higher frequencies ($\beta = 6$), heat transfer is reduced in the range of $A_f < 1$, while it is enhanced significantly in

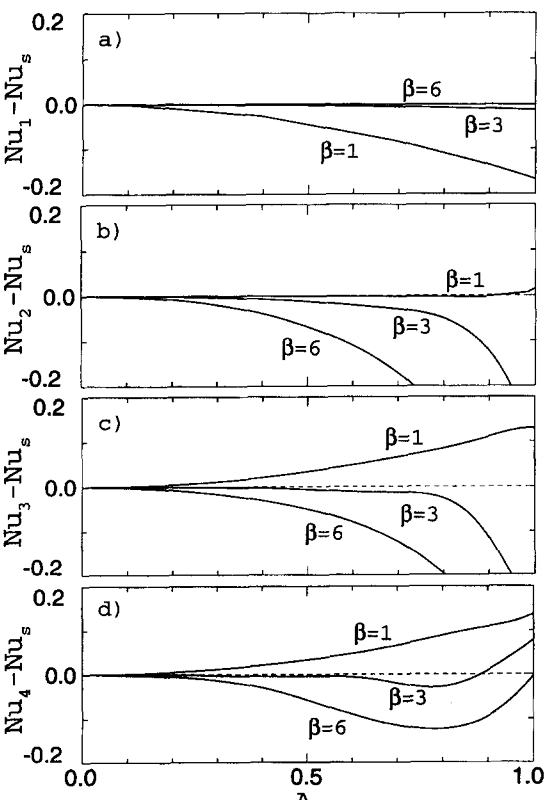

Fig. 1. Comparison of Nusselt numbers for $\beta = 1, 3$ and 6: $0 \leqslant A_f < 1$.

Table 1. Comparison of different Nusselt numbers with the experiment of Kim [9]: $\beta = 6.18$, $Re = 385$ and $Pr = 0.7$

A_f	Nu_1	Nu_2	Nu_3	Nu_4	Experiment
1.05	4.36	—	—	4.37	4.32±0.1
2.05	4.35	—	—	5.08	5.14±0.1

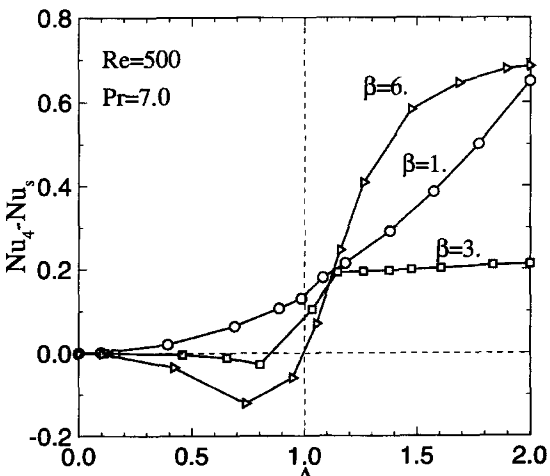

Fig. 2. Prediction of $Nu_4 - Nu_s$ for $\beta = 1$, 3 and 6: $0 \leqslant A_f \leqslant 2.0$.

the higher-amplitude range ($A_f > 1$). However, the Nusselt number tends to be independent of the magnitude of amplitude as A_f increases further. This is consistent with the experimental findings of Hwang and Dybbs [4]. For the lower-frequency range ($\beta = 1$), $Nu_4 - Nu_s$ increases monotonically with increasing A_f. A closer inspection of Fig. 2 discloses that, as A_f increases, the influence of pulsation on heat transfer at $\beta = 3$ is insignificant.

In order to look into the effects of β on heat transfer for $A_f \geqslant 1$ in more detail, the distributions of $Nu_4 - Nu_s$ are demonstrated in Fig. 3. Three large-amplitudes ($A_f = 1.2, 1.5$ and 1.8) are selected for comparison. Except for the local minimum around $\beta \approx 2.5$, which is related to the small influence of pulsation at $\beta = 3$ in Fig. 2, the heat transfer increases with increasing frequency. However, β has a weak influence on heat transfer in the low-frequency area ($0 < \beta \leqslant 0.5$).

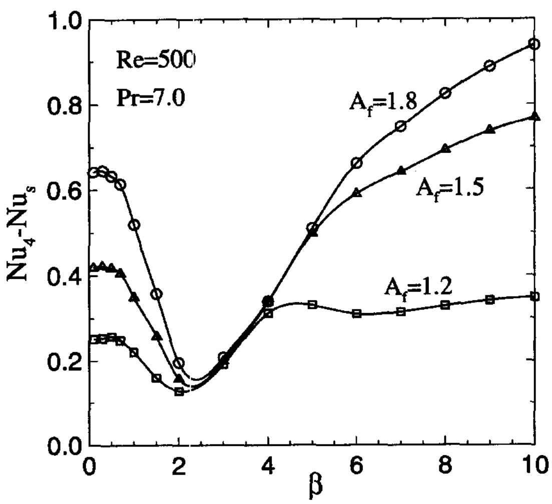

Fig. 3. Prediction of $Nu_4 - Nu_s$ for $A_f = 1.2$, 1.5 and 1.8: $0 < \beta \leqslant 10$.

5. CONCLUSION

Many versions of the Nusselt number have been tested to clarify the conflicting results in the heat transfer characteristics for pulsating flow in a pipe. An improved version of the Nusselt number (Nu_4) was proposed in the present study, which was shown to be in close agreement with available measurements. The main emphasis was placed on the large amplitude of the pulsation flowrate ($A_f \geqslant 1$). For a small amplitude ($0 < A_f < 1$), both heat transfer enhancement and reduction were detected depending on the pulsation frequency (β). Of much importance is the fact that, for a large amplitude ($A_f \geqslant 1$), the heat transfer due to pulsation is always augmented.

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