Transient Radiative Heat Transfer in a Cubic Participating Medium Subjected to a Diffuse Square Pulse Train

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Abstract

Three-dimensional numerical computations were carried out to understand the overlap effect of the transmitted pulses in a cubic participating medium subjected to a diffuse square pulse train using the transient discrete ordinates method with the pulse boundary condition. In concrete terms, the effects of the pulse width and pulse train interval on the divergence of radiative heat flux were scrutinized for both optically thin and thick media. The transient characteristics of radiative heat transfer when the transmitted pulses were detected as distinct individual signals or as a single signal due to the overlap effect were elucidated by visualizing the transmitted pulses in a 3D medium continuously.

Nomenclature

$c$ speed of light in medium
$G$ incident radiation
$I$ radiation intensity
$I_p$ radiation intensity at the node of a control volume
$L$ length
$N$ angular discrete order in $S_N$ approximation
$n$ number of angular discretization
$Q$ net radiative heat flux
$q$ radiative heat flux
$r$ position vector
$S$ source term
$s$ Discrete ordinates direction
$T$ transmittance
$t$ time
$t_d$ pulse train interval between two successive pulses
$t_p$ pulse width
$w$ angular weight
$x,y,z$ Cartesian coordinates
$\Phi$ scattering phase function
$\gamma$ weighting factor
$\xi,\eta,\mu$ direction cosines
$\rho$ diffuse reflectivity
$\sigma_a$ absorption coefficient
$\sigma_t$ extinction coefficient
$\sigma_s$ scattering coefficient
$\omega$ scattering albedo
$*$ dimensionless quantity

Introduction

Computed tomography using X-rays and magnetic resonance imaging (MRI) is currently used extensively to obtain imaging information about morphology inside living bodies. In this context, the development of optical computed tomography (Optical-CT) using ultra-short pulsed lasers has been proceeding vigorously because Optical-CT is expected to obtain imaging information about physiology as well as morphology inside living bodies [Hebden et al., 2001; Quan and Guo, 2004; Yamada, 1995]. To realize fully-functional Optical-CT, the clarification of the transient radiative transfer characteristics of scattered signals induced by the interaction of pulsed light in highly scattering and weakly absorbing biological tissues is needed.

The accurate modeling of ultrafast radiative transfer in highly scattering media such as biological tissues is important to understand the behaviour of scattered signals in a medium. Therefore, various numerical methods have been developed, mainly in two categories of stochastic and deterministic methods. The Monte Carlo method would be a stochastic approach [Guo et al., 2002; Wu, 2009] while the discrete ordinates method (DOM) [Guo and Kumar, 2002; Hunter and Guo, 2012] and finite volume method [Chai et al, 2004; Hunter and Guo, 2011] are deterministic approaches. Among them, the DOM has been one of the most widely applied methods in radiative transfer modelling because it is algorithmically simple and compatible with CFD codes. Therefore, the DOM is adopted in the present study.

Investigations of the transient radiative transfer characteristics in highly scattering media have been focused on the transport of a single ultra-short pulse [Guo and Kumar, 2001; Wu and Ou, 2002; Guo and Kim, 2003; Mishra et al, 2006 and 2011; Okutucu and Yener, 2006]. However, in reality, continuous pulse trains are usually applied. Therefore, the present authors conducted a complete transient three-dimensional simulation using the DOM and Duhamel’s superposition theorem to study the interaction of diffuse irradiation of a square pulse train in a cube of highly scattering medium [Akamatsu and Guo, 2011]. In that paper, we concluded that the overlap effect of the pulse train depends on the pulse train interval between two successive pulses as well as the magnitude of the pulse broadening in the response subjected to a single pulse; and is independent of the pulse width. That is, the magnitude of the overlap effect depends on the compatibility between the pulse interval and the broadening of a single pulse. These conclusions were made by scrutinizing the temporal distributions of transmittance in a cubic participating medium subjected to a train of pulses of various pulse widths and various time intervals between pulses. In the present study, visualization of the divergence of radiative heat flux in a cubic participating medium subjected to a train of pulses is carried out via the transient discrete ordinates method with the pulse boundary...
condition in order to more thoroughly understand the overlap effect of transmitted pulses.

**Governing equations**

The model system used in the present computations is shown in Figure 1. The cubic homogeneous medium was assumed to be a non-emitting, strongly scattering and weakly absorbing medium. The wall at \( x = 0 \) was heated instantaneously and isothermally at time \( t = 0 \), and the heating was periodically changed just like a train of short pulses. Detector A was installed at \( x^* = 1, y^* = z^* = 0.5 \), i.e., \( (x, y, z) = (L, 0.5L, 0.5L) \) in the dimension, for capturing the transmitted pulse. The transient radiative transfer in a cube was computed directly from the pulse boundary condition using DOM. The pulse train interval between two successive pulses was computed directly from the pulse boundary condition using

\[
\sum_{t=1}^{5} \xi^i w' I^i = 0, \quad Q = \frac{\sum_{l} \xi^i w' I^i}{1}, \quad G = \sum_{l} \xi^i w' I^i. 
\]

The divergence of the total heat flux for the non-emitting medium is obtained by

\[
\nabla \cdot \mathbf{q} = -(\omega - \sigma) \cdot G
\]

The transient transmittance at the wall of \( x = L \) is defined as

\[
T(x^* = 1, y^* = z^* = 0.5, t^*) = Q_w(x^* = 1, y^* = z^* = 0.5, t^*)
\]

**Numerical scheme**

The control volume approach is used for the spatial discretization to solve equation (1). The cube is divided into small control volumes by \( M_x \times M_y \times M_z \) meshes. Figure 2 shows a control volume in two-dimensional coordinates.

In each control volume, equation (1) is discretized temporally and spatially. The final discretization equation for the cell intensity in a generalized form \([1, 4]\) becomes

\[
I^i_v = \frac{1}{c \Delta t} I^i_{p} + \sigma_S S^i_{v} + \frac{1}{\gamma^i_{xx} \Delta x} I^i_{xa} + \frac{1}{\gamma^i_{yx} \Delta y} I^i_{ya} + \frac{1}{\gamma^i_{zz} \Delta z} I^i_{za},
\]

where \( \rho \) is the diffuse reflectivity of the wall surface. Similarly, the relationships for the remaining five walls can be set up. Only in the case of the wall \( x = 0 \) is a periodic change of the unity blackbody emissive power assumed. In the present study, \( \rho = 0 \); i.e., the sidewalls are black and absorb all incident radiation.

Once the intensities have been determined, the net radiative heat flux in the \( x \) direction and the incident radiation \( G \) are evaluated from

\[
Q_x = \sum_{l} \xi^i w' I^i, \quad G = \sum_{l} \xi^i w' I^i.
\]
where $I_p$ is the intensity at the node of the control volume; $I_p^0$ is the intensity at the previous time step; and $I_{ud}^\gamma$, $I_{sd}^\gamma$, and $I_{ud}^\gamma$ are the radiation intensities on the upstream surface in the $\gamma$ direction as shown in Figure 2. In the present study the positive scheme proposed by Lathrop [11] is applied to determine the values of weighting factors $\gamma_1$, $\gamma_2$, and $\gamma_3$. With the nodal intensity obtained from Eq. (7), the unknown radiation intensities on the downstream surface in the same direction are computed as follows:

$$I_p = \gamma_1I_{ud} + (1-\gamma_1)I_{ud}^0 = \gamma_1I_{sd} + (1-\gamma_1)I_{sd}^0, \quad (8)$$

where $I_{ud}^\gamma$, $I_{sd}^\gamma$, and $I_{ud}^\gamma$ are the radiation intensities on the downstream surface in the $\gamma$ direction as shown in Figure 2. The derivation of equation (8) is well described by Modest [15] for the steady-state situation. In equation (7), the traveling constant $c\Delta t$ should not exceed the control volume spatial step; i.e., $c\Delta t < \min[\Delta x, \Delta y, \Delta z]$. This is because a light beam always travels at the speed of light $c$ in the medium. Hence, the following condition is imposed to eliminate the numerical diffusion:

$$\Delta t < \min[\Delta x^*, \Delta y^*, \Delta z^*] \quad (9)$$

The following dimensionless variables are defined as

$$t^* = \frac{ct}{L}, \quad x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}, \quad z^* = \frac{z}{L}. \quad (10)$$

In our previous numerical investigation [1], we examined the influences of the spatial grid and time step on the temporal profiles of reflectance and transmittance to minimize the effects of false radiation propagation and numerical diffusion for solving ultrafast radiative transfer in a highly scattering medium, and the optimum spatial grid and time step were determined. The present numerical computations adopted the spatial grid and time step determined in the previous study. Namely, the spatial grid number and the time step were $49 \times 49 \times 49$ and $\Delta t^* = 0.006$, respectively.

Results and discussion

Figure 3 shows temporal distributions of the divergence of radiative heat flux in the cubic medium for $\sigma_r L = 0.1$, $\omega = 0.9$ at $t^* = 8.1$ when the magnitude of $\text{div} q$ reaches its maximum by the irradiation of five identical square pulses with $t_p^* = t_0^* = 0.9$. The $\text{div} q$ takes the maximum value at $t^* = 8.1$, i.e., $t^* = t_0^* \times 5 + t_p^* \times 4$. This time instant is plotted by the red solid circle as $t_p^*$ in Figure 3. The $\text{div} q$ is drawn in the volume $1/49 \leq x^*, y^*, z^* \leq 48/49$ and at intervals of 0.001 between 0 and 0.2. The divergence of radiative heat flux translates to the energy deposition rate. It is seen that the depth of the energy deposition reaches the centre of the cube from the wall of $x^* = 0$ subjected to diffuse square pulse train; the magnitude of $\text{div} q$ at the central part is larger than that of the circumferential part in the $X-Z$ planes at the same $x^*$ position, and decreases gradually with the increase of the $x^*$ position.

Figure 4 shows contour plots of the divergence of radiative heat flux in the cubic medium for $\sigma_r L = 0.1$, $\omega = 0.9$ at $t^* = 8.1$ when the magnitude of $\text{div} q$ reaches its maximum by the irradiation of five identical square pulses with $t_p^* = t_0^* = 0.9$. The $\text{div} q$ takes the maximum value at $t^* = 8.1$, i.e., $t^* = t_0^* \times 5 + t_p^* \times 4$. This time instant is plotted by the red solid circle as $t_p^*$ in Figure 3. The $\text{div} q$ is drawn in the volume $1/49 \leq x^*, y^*, z^* \leq 48/49$ and at intervals of 0.001 between 0 and 0.2. The divergence of radiative heat flux translates to the energy deposition rate. It is seen that the depth of the energy deposition reaches the centre of the cube from the wall of $x^* = 0$ subjected to diffuse square pulse train; the magnitude of $\text{div} q$ at the central part is larger than that of the circumferential part in the $X-Z$ planes at the same $x^*$ position, and decreases gradually with the increase of the $x^*$ position.
Figure 5 shows contour plots of divergence of radiative heat flux in the cubic participating medium and Y-Z plane of \(x^* = 0.5\) at (a) \(t^* = 6.702\) and (b) \(t^* = 7.584\) for \(\sigma_r L = 0.1, \omega = 0.9\). The \(\text{divq}\) reaches its maximum in Y-Z planes of \(x^* = 5\) at \(t^* = 6.702\), and this time instant is plotted by the blue circle as \(t_{\text{min}}^*\) in Figure 3. On the other hand, the \(\text{divq}\) takes its minimum value in Y-Z planes of \(x^* = 0.5\) at \(t^* = 7.584\), and this time instant is plotted by the blue circle as \(t_{\text{min}}^*\) in Figure 3. In Figure 5(a), the energy of the fourth irradiated diffuse square pulse just passes through the centre of the cube. In the Y-Z plane, \(\text{divq}\) reaches its maximum magnitude of 0.0066 at the central region. The magnitude of the \(\text{divq}\) becomes gradually smaller toward the circumferential region. Figure 5(b) shows contour plots of divergence of radiative heat flux when the hot wall at \(x^* = 0\) is subjected to the fifth diffuse square pulse. The energy of the fourth irradiated diffuse square pulse passes through the cubic participating medium completely. In the Y-Z plane, the distribution of the \(\text{divq}\) is uniform and its magnitude is zero since the energy of the fifth irradiated diffuse square pulse does not reach the central region of the cube.

Figure 6 shows temporal distributions of the divergence of radiative heat flux at \(x^* = y^* = z^* = 0.5\) and transmittance at \(x^* = 1, y^* = z^* = 0.5\) for \(\sigma_r L = 0.1, \omega = 0.9\) when the hot wall at \(x^* = 0\) is subjected to five square pulses with dimensionless pulse width \(t_{p^*} = 0.21\) and pulse train interval \(t_{d^*} = t_{p^*}\). A diffuse square pulse train with the pulse width of about 0.23 times that shown in Figure 5 was imposed on the cubic participating medium. Although five distinct signals were detected like those in Figure 3, these signals did not take the value of zero between successive pulses. That is, the responses of the transmitted pulses overlapped. The period and amplitude of \(\text{divq}\) are 0.42 and 0.0031. On the other hand, those of transmittance are 0.42 and 0.086. These periods are two times the width of the irradiated pulse.

Figure 7 shows contour plots of divergence of radiative heat flux in the cubic participating medium for \(\sigma_r L = 0.1, \omega = 0.9\) at \(t^* = 1.89\) when the magnitude of \(\text{divq}\) reaches its maximum by the irradiation of five square pulses with \(t_{p^*} = t_{d^*} = 0.21\). The \(\text{divq}\) takes the maximum value at \(t^* = 1.89\), i.e., \(t^* = 1.89 \times 5 + t_{p^*} \times 4\). This time instant is plotted by the red solid circle as \(t_{\text{max}}^*\) in Figure 6. The \(\text{divq}\) is drawn at intervals of 0.001 between 0 and 0.02. By the decreases of pulse width and pulse interval, the two energy distributions of the fifth and fourth irradiated pulses can be observed clearly in the cubic participating medium.

Figure 5. Contour plots of the divergence of radiative heat flux in the cubic participating medium and Y-Z plane of \(x^* = 0.5\) at (a) \(t^* = 6.702\) and (b) \(t^* = 7.584\) for \(\sigma_r L = 0.1, \omega = 0.9\).

Figure 6. Temporal distributions of the divergence of radiative heat flux at \(x^* = y^* = z^* = 0.5\) and transmittance at \(x^* = 1, y^* = z^* = 0.5\) for \(\sigma_r L = 0.1, \omega = 0.9\) when the hot wall at \(x^* = 0\) is subjected to five square pulses with dimensionless pulse width \(t_{p^*} = 0.21\) and pulse train interval \(t_{d^*} = t_{p^*}\).

Figure 7. Contour plots of divergence of radiative heat flux in a cubic participating medium for \(\sigma_r L = 0.1, \omega = 0.9\) at \(t^* = 1.89\) when the magnitude of \(\text{divq}\) becomes the maximum by the irradiation of five square pulses with \(t_{p^*} = t_{d^*} = 0.21\).
Figure 8 shows contour plots of divergence of radiative heat flux in a cubic participating medium and Y-Z plane of $x^* = 0.5$ at (a) $t^* = 1.950$ and (b) $t^* = 2.160$ for $\sigma_r L = 0.1$, $\omega = 0.9$.

Figure 9 shows temporal distributions of the divergence of radiative heat flux at $x^* = 1$, $y^* = z^* = 0.5$ and transmittance at $x^* = 1$, $y^* = z^* = 0.5$ for $\sigma_r L = 0.1$ (optically thin medium), $\omega = 0.9$ when the hot wall at $x^* = 0$ is subjected to five square pulses with dimensionless pulse width $t_p^* = t_d^* = 0.06$. The divergence of radiative heat flux at $t^* = 0.54$, i.e., $t^* = t_p^* x 5 + t_d^* x 4$. This time instant is plotted by the red solid circle as $t_p^*$ in Figure 9. The divergence of radiative heat flux is plotted by the blue circle as $t_q^*$ in Figure 6. On the other hand, the divergence of radiative heat flux reaches its maximum by the irradiation of five square pulses with dimensionless pulse width $t_p^* = 0.06$ and pulse train interval $t_p^* = t_d^*$.

Figure 10 shows contour plots of the divergence of radiative heat flux in the cubic participating medium for $\sigma_r L = 0.1$, $\omega = 0.9$ at $t^* = 0.54$ when the magnitude of $\text{div} q$ becomes the maximum by the irradiation of five square pulses with $t_p^* = t_d^* = 0.06$. The magnitude of $\text{div} q$ takes the maximum value at $t^* = 0.54$, i.e., $t^* = t_p^* x 5 + t_d^* x 4$. This time instant is plotted by the red solid circle as $t_p^*$ in Figure 9.

Figure 9. Temporal distributions of the divergence of radiative heat flux at $x^* = 1$, $y^* = z^* = 0.5$ and transmittance at $x^* = 1$, $y^* = z^* = 0.5$ for $\sigma_r L = 0.1$, $\omega = 0.9$ when the hot wall at $x^* = 0$ is subjected to five square pulses with dimensionless pulse width $t_p^* = 0.06$ and pulse train interval $t_p^* = t_d^*$.

Figure 10. Contour plots of the divergence of radiative heat flux in the cubic participating medium for $\sigma_r L = 0.1$, $\omega = 0.9$ at $t^* = 1.002$ for $\sigma_r L = 0.1$, $\omega = 0.9$. The divergence of radiative heat flux reaches its maximum value in Y-Z planes of $x^* = 0.5$ at $t^* = 1.002$ and this

Figure 11 shows contour plots of the divergence of radiative heat flux in the cubic participating medium and Y-Z plane of $x^* = 0.5$ at $t^* = 1.002$, and this
time instant is plotted by the blue solid circle at \(t_{\text{max}}*\) in Figure 9. In Figure 11, it is noted that the single energy distribution only exists near the centre of the cube. Although the some energy distributions are observed near the plane subjected to pulse trains as shown in Figure 10, it is considered that those energy distributions totally overlap as the energy of the transmitted pulse is transferred in the \(x*\) direction. In the Y-Z planes, the \(\text{div} q\) reaches its maximum magnitude of about 0.0034 at the central region.

Figure 11. Contour plots of the divergence of radiative heat flux in the cubic participating medium and Y-Z plane of \(x*=0.5\) at \(t*=1.002\) for \(\sigma_s L=0.1\), \(\rho=0.9\).

Figure 12 shows temporal distributions of the divergence of radiative heat flux at \(x*=y*=z*=0.5\) and the transmittance at \(x*=1\), \(y*=z*=0.5\) (optically thick medium), \(\omega=0.9\) when the hot wall at \(x*=0\) is subjected to five square pulses with dimensionless pulse width \(t_p*=0.06\) and pulse train interval \(t_d*=t_p*\). The optical thickness is 100 times those shown in Figures 3-11. The pulse width of the irradiated diffuse square pulse train is the same as those shown in Figures 9-11. It is also noted that the output signal is detected as a single pulse in spite of the irradiation of five identical square pulse trains. However, the periods of \(\text{div} q\) and the transmittance are much longer than those for the optically thin medium shown in Figure 9. The magnitude of amplitude of \(\text{div} q\) is about 14 times and that of the transmittance is about 1/140 times those of the optically thin medium shown in Figure 9. The period and amplitude of \(\text{div} q\) are about 4 and 0.049. On the other hand, those of transmittance are about 4.9 and 7.33\(\times10^{-5}\). These periods are about 67-82 times the width of the irradiated pulse.

Figure 13 shows contour plots of the divergence of radiative heat flux in the cubic participating medium and Y-Z plane of \(x*=0.5\) at \(t*=1.128\) for \(\sigma_s L=10\), \(\rho=0.9\). The \(\text{div} q\) reaches its maximum value in Y-Z plane of \(x*=0.5\) at \(t*=1.128\), and this time instant is plotted by the blue solid circle at \(t_{\text{max}}*\) in Figure 12. In the left-hand-side figure, the \(\text{div} q\) is drawn at intervals of 0.1 between 0 and 2. On the other hand, in the right-hand-side figure, it is drawn at intervals of 0.001 between 0 and 0.02 like those for the optically thin medium shown in Figures 3-11. It is noted from the increase in the extinction coefficient that the increase in the energy deposition rate is significant. It is also considered that the increase of the energy deposition rate causes the pulse broadening.

Conclusions

The visualization of the divergence of radiative heat flux in a cubic participating medium subjected to a train of diffuse square pulses was carried out via the transient discrete ordinates method with the pulse boundary condition in order to thoroughly understand the overlap effect of the transmitted pulse discovered in our previous work. The transient behaviour of the transmitted pulse when the transmitted pulses were detected as distinct individual signals and as a single signal due to the overlap effect was revealed by visualizing the transient energy deposition rate inside the 3D medium. The overlap effect of the transmitted pulse train strongly depended on the pulse train interval as well as the optical thickness.

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References


