Fresnel Effect in Radiation Transfer in Biological Tissues

Kyunghan Kim and Zhixiong Guo*
MAE Department, Rutgers University, Piscataway, NJ 08854, USA

Yong Jin
Bioarray Solutions, Piscataway, NJ, USA

Sunil Kumar
Polytechnic University, Brooklyn, NY, USA

In this paper, we extended the Discrete Ordinary Method (DOM) to incorporate Fresnel’s boundary in laser radiation transport in biological tissues. A three-dimensional rectangular model is constructed for demonstrating the usage of the DOM. The present method can treat both the incident laser intensity and radiation intensity from the walls as two components, i.e., a diffusion part and a specular part. Reflectivity at the tissue/air interface is calculated by the use of Snell’s law and Fresnel’s equation. The radiation fields, including the radiative heat flux, the radiation deposition rate and the irradiation are calculated. Influence of Fresnel’s boundary is investigated by comparing the predictions between specular and diffuse boundary models. The present results reveal that the strength of the transmitted or reflected signals is larger in the case of specular boundary than in diffuse case. Fresnel’s reflection should be considered in photon transport and light imaging wherever tissue/air interfaces are present.

1. Introduction

Short-pulsed laser interaction and propagation in biological tissues have attracted a great deal of interest in recent years because of its applications in biomedical treatment and diagnostics. The transient nature of radiation transport in such applications is introduced via the inclusion of time derivative in the radiative transfer equation that accounts for the speed of radiation propagation. Some unique features associated with the transient radiation are being exploited and becoming well known, such as the broadening of reflected and transmitted pulses after a laser pulse passes through a scattering medium and the temporal signature that can be exploited to reconstruct images and to enhance image contrast as compared with CW laser irradiation.

The discrete ordinates method (DOM) has become one of the most popular methods for solving Boltzmann transport. In the present study the DOM is formulated for time-dependent radiative transfer in anisotropically scattering, absorbing, and emitting media in three-dimensional rectangular enclosures. Fresnel boundary is induced at the tissue/air interface owing to the difference of refractive indices between air and biological tissue. Reflectivity at the tissue/air interface is calculated by the use of Snell’s law and Fresnel’s equation.

2. Mathematical Model

The transient radiative transfer equations in discrete ordinates form for laser transport is formulated as

\[
\frac{1}{c} \frac{\partial I}{\partial t} + \sum_{j=1}^{\infty} w_j \Phi^j \frac{\partial I}{\partial \xi} + \sigma_{\alpha} I = \sigma_S S_l^l
\]

where \(\Phi^j\) represents scattering phase function, and \(S_l^l\) is the source contribution of collimated irradiation such as laser incidence. A quadrature set of \(n\) discrete ordinates with the appropriate angular weight \(w_j\) is used in the “\(S_N\)-approximation” of the source term.

The boundaries can reflect incident radiation. The reflection can be considered both diffuse and specular effects as shown in Fig. 1. When the Fresnel boundary is concerned, the reflection is specular. Considering a radiation from tissue hits at the interface of tissue/air, Snell’s law is obeyed, i.e., \(n_T \sin \theta_i = n_{air} \sin \theta_r\). Here, the incident angle is \(\theta_i\) and the refraction angle is \(\theta_r\). Since the refractive index of the tissue \(n_T\) is larger than the refractive index of air \(n_{air}\), total reflection occurs at the boundaries when \(\theta_i\) is not less than a critical angle \(\theta_{cr} = \sin^{-1}(n_{air}/n_T)\). When \(\theta_i < \theta_{cr}\), the specular reflectivity is calculated by the Fresnel equation as

\[
\rho = \frac{1}{2} \left[ \frac{\tan^2(\theta_i - \theta_{cr}) + \sin^2(\theta_i - \theta_{cr})}{\tan^2(\theta_i + \theta_{cr}) + \sin^2(\theta_i + \theta_{cr})} \right].
\]

The diffuse intensity at wall is

\[
I_{d} = \epsilon_n I_{in} + \rho \sum_{\xi < 0} w_j |I_j| |
\]
while the specular intensity at wall is

\[ I_w^l = \rho_w^l I^l. \]

(5)

Here \( \rho_w^l \) is the diffuse reflectivity of the interface and \( \rho_s^l \) is the specular reflectivity that is calculated by the Fresnel equation. The treatment of laser pulse has been described by Guo in detail.

3. Results and Discussion

A cubic tissue phantom with side length of 10 mm is considered. The optical properties are \( \sigma_a = 0.01 \text{ mm}^{-1} \) and \( \sigma_s = 1.00 \text{ mm}^{-1} \). Scaling law is adopted for handling anisotropic scattering. Uniform control volume meshes are adopted. The spatial step is \( \Delta x = \Delta y = \Delta z = 0.04 \text{ mm} \). The time resolution is \( \Delta t = 0.3 \text{ ps} \).

The influence of Fresnel boundary condition is demonstrated in Fig. 2, for which two different media are in contact at the tissue/air interface. The refractive index of air is 1.0, while the one of tissue is 1.4. Due to this abrupt change of refractive index at the interface, the Fresnel boundary condition must be considered. In order to study the influence of boundary condition, we compare the temporal reflectance and transmittance profiles at two extremely cases, i.e., purely specular and purely diffuse. Fresnel’s equation is used to predict the reflectivity. It is seen that the predicted magnitude of reflectance or transmittance under specularly reflecting condition is greater than that under diffusely reflecting condition. Also the peak position for specular boundary shifts to a later time.

Comparison of divergence of radiative heat flux in Y-Z planes between diffusely reflecting boundary and specularly reflecting boundary is disclosed in Fig. 3. Here shows only the time instant at \( t = 120 \text{ ps} \). At the three selected different planes, the divergence of radiative heat flux is larger in specularly reflecting case than in diffusely reflecting case. At the center area of planes \( x = 2.5 \text{ and } 5.0 \text{ mm} \), the divergence of radiative heat flux for specular boundary is nearly twice that for diffuse boundary. At the boundary vicinity, however, the divergence of radiative heat flux for the specular boundary is almost one order of magnitude larger than that for the diffuse boundary. At the plane of \( x = 7.5 \text{ mm} \), the difference between the predictions of these two boundary conditions reduces. The results in Figs. 2 and 3 reveal that the radiation transfer boundary is an important factor in the accurate prediction of photon transport in tissue, particularly at the boundary territory. In the popularly adopted diffusion approximation for photon migration in tissues, however, the Fresnel boundary condition can hardly be incorporated.

4. Conclusions

The discrete ordinates method is developed to study transient radiative transfer in three-dimensional scattering, absorbing, and emitting media subject to Fresnel boundary condition. The Fresnel boundary effect with either specular reflection or diffuse reflection at the tissue/air interface is investigated. The reflectance, transmittance and divergence of radiative heat flux inside the tissue in the specularly reflecting boundary are greater than those in the diffusely reflecting boundary. The adequate incorporation of Fresnel’s reflection is very important.

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References


Fig. 1. Reflection and refraction at the tissue/air interface.

Fig. 2. Difference of temporal reflectance and transmittance the predictions of between specularly reflecting boundary and diffusely reflecting boundary: 1.2 – reflectance; 6.7 – transmittance.

Fig. 3. Comparison of divergence of radiative heat flux in Y-Z planes between diffusely reflecting boundary and specularly reflecting boundary.