IMPROVEMENT OF COMPUTATIONAL TIME IN RADIATIVE HEAT TRANSFER OF THREE-DIMENSIONAL PARTICIPATING MEDIA USING THE RADIATION ELEMENT METHOD

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ABSTRACT. The radiation element method by ray emission model (REM^2) has been improved by using the stabilized bi-conjugate gradient (BiCGSTAB) method and reduction of the size of equations in order to reduce computational time. This improved method was applied to analyze radiative heat transfer in arbitrary three-dimensional participating media and enclosures. The accuracy of the improved method was evaluated by comparing its predictions with the Monte Carlo and the YIX solutions. And the method was used to calculate radiative heat transfer in the boiler furnace. Total CPU time to calculate the radiative heat transfer for a model comprised of 3211 elements was reduced to 1/22 of that by the previous numerical method using a decomposition method.

NOMENCLATURE

$\mathbf{F}$	View factor matrix
$F_{i,j}$	View factor from element i to j
$\mathbf{I}$	Unit matrix
N, n_p	Total number of elements and number of partition
N_c	Density of carbon particles
q_X	Net heat flux for surface element or divergence of heat flux for volume element
$\mathbf{Q}$	Heat transfer rate vector
Q_G, Q_J	Heat transfer rate of irradiation energy and diffuse radiosity
Q_T, Q_X	Heat transfer rate of emissive power and net heat generation
Ω	Albedo or reflectivity
Subscripts	
i	Element i
j	Element j
λ	Spectral value
Superscripts	
A	Absorption
D	Diffuse scattering
R	Radiation
S	Specular

INTRODUCTION

It is important to analyze radiative heat transfer in high temperature objects, such as boiler furnaces and heaters for heating semiconductors, because radiative heat transfer is the dominant mode of energy transfer in these equipments. Recently, many methods have been proposed to predict radiative heat transfer. For example, Farmer et al. presented the Monte Carlo method [1] and Hsu et al. presented a YIX method [2] for predicting radiative heat transfer. Tong and Skocypec compared several numerical methods, such as Monte Carlo method, YIX method, and Generalized Zonal method and summarized their solutions for three-dimensional anisotropic scattering media [3]. Recent improvements in computer technology enable analysis of radiative heat transfer by using many calculation meshes and elements. However, such analysis requires a long computational time. Therefore, it is necessary to develop a fast method of calculating radiative heat transfer.

Many methods have been proposed to reduce CPU time for predicting radiative heat transfer. For example, Yang et al. developed a radiative heat-ray method using the READ (radiative energy absorption rate distribution) [4]. Maruyama et al. improved the zone method and developed the radiation element method by ray emission model (REM^2) and predicted radiative heat transfer in gray homogeneous media with complex configuration [5]. Furthermore, REM^2 was developed to take the nongray and anisotropic scattering media by Guo and Maruyama [6]. This method is much faster than the Monte Carlo method. However, the zone method and the REM^2 need a long CPU time to calculate radiative exchange.

The REM^2 have to solve linear equations in the analyses of radiative heat transfer. However, it also took a long CPU time for calculations with a large number of elements. In previous studies, the REM^2 have solved the linear equations by a direct method such as decomposition method, and it takes a CPU time that is proportional to the third power of the number of the elements. On the other hand, iterative methods such as the stabilized bi-conjugate gradient (BiCGSTAB) method have been known as efficient algorithms to solve unsymmetrical linear equations [7]. The BiCGSTAB method was developed to solve unsymmetrical linear systems while avoiding the irregular convergence patterns of the Conjugate Gradient method. The BiCGSTAB method is generally more efficient than the direct methods, such as the *LU* decomposition method, in solving large linear equations.

In this paper, the size of linear equations to solve in the REM^2 was reduced. Then these linear equations were solved by the BiCGSTAB method, resulting in a much shorter CPU time for predicting radiative heat transfer.

METHOD OF ANALYSIS

We consider a nongray, anisotropic scattering, absorbing, and emitting medium. To simplify the problem, the following assumptions are introduced. (1) Each element has constant temperature, refractive index, and heat generation rate per unit volume. (2) The scattered radiation is distributed uniformly over the element. Under these assumptions, Maruyama introduced the average diffuse radiant intensity, I_λ^D, and an effective radiation area, A_λ^R. Then Maruyama developed the radiation element method by ray emission model (REM^2) [5]. I_λ^D is similar to the diffuse radiosity that was used for radiation transfer of arbitrary diffuse and specular surfaces. A_λ^R of element i is defined as following expression.

$$A_{i,\lambda}^R = \frac{1}{\pi}\int_{4\pi} A_i(\hat{s})\left[1-\exp\left(-\beta_{i,\lambda}\bar{S}_i(\hat{s})\right)\right]d\omega \quad (1)$$

where $A_i(\hat{s})$ is an area projected onto the surface normal to $\hat{s}$, β_λ is the extinction coefficient, and $\bar{S}(\hat{s})$ is an averaged thickness of the radiation element in the direction $\hat{s}$, respectively. In the REM^2, the rate of spectral radiation energy emitted and isotropically scattered by the radiation element can be expressed in a generalized form as

$$Q_{J,i,\lambda} = A_{i,\lambda}^R\left(\varepsilon_{i,\lambda}E_{b,i,\lambda} + \Omega_{i,\lambda}^D G_{i,\lambda}\right) \quad (2)$$

where $\varepsilon_{i,\lambda} = 1 - \Omega^D_{i,\lambda} - \Omega^S_{i,\lambda}$, $E_{b,i,\lambda} = \pi I_{b,i,\lambda}$, $G_{i,\lambda} = \pi I^D_{i,\lambda}$, and $Q_{J,i,\lambda}$ is the diffuse radiation transfer rate. The net rate of heat generation can be derived from the heat balance on the radiation element as follows.

$$Q_{X,i,\lambda} = A^R_{i,\lambda}\varepsilon_{i,\lambda}\left(E_{b,i,\lambda} + G_{i,\lambda}\right) \tag{3}$$

If the system is consisted of N volume and surface elements, then Eqs.(2) and (3) can be rewritten as

$$Q_{J,i,\lambda} = Q_{T,i,\lambda} + \sum_{j=1}^{N} F^D_{j,i,\lambda} Q_{J,i,\lambda} \qquad\qquad Q_{X,i,\lambda} = Q_{T,i,\lambda} - \sum_{j=1}^{N} F^A_{j,i,\lambda} Q_{J,i,\lambda} \tag{4}$$

where $Q_{T,i,\lambda} = A^R_{i,\lambda}\varepsilon_{i,\lambda}E_{b,i,\lambda}$, and the absorption view factor, $F^A_{i,j,\lambda}$, and diffuse scattering view factor, $F^D_{i,j,\lambda}$, are introduced. When $F_{i,j,\lambda}$ is denoted as matrix $\mathbf{F}_\lambda$ and $Q_{i,\lambda}$ is denoted as vector $\mathbf{Q}_\lambda$, diffuse radiative heat transfer rates for all elements are obtained as follows.

$$\mathbf{Q}_{J\lambda} = \mathbf{Q}_{T\lambda} + \mathbf{F}^D_\lambda \mathbf{Q}_{J\lambda} \tag{5}$$

In the previous REM[2], $Q_{J,i,\lambda}$ is eliminated from Eq.(4), and the relation between $Q_{T,i,\lambda}$ and $Q_{X,i,\lambda}$ is obtained. Then $Q_{T,i,\lambda}$ for each radiation element is given as a boundary condition, and the unknown $Q_{X,i,\lambda}$ can be attained. However, during the procedure of eliminating $Q_{J,i,\lambda}$ by the *LU* decomposition method, a long CPU time was consumed. Therefore, to reduce a CPU time, the size of equations was reduced and these equations were solved by BiCGSTAB method, which is known as an efficient method.

When the albedo or diffuse reflectivity of element i, $\Omega^D_{i,\lambda}$, is 0, or $F^D_{j,i,\lambda}$ is 0, $Q_{J,i,\lambda}$ is equal to $Q_{T,i,\lambda}$ from the relation in Eq. (5). When the system has n_1 elements with nonzero albedo and diffuse reflectivity, and the remaining n_2=N-n_1 elements are $\Omega^D_{i,\lambda} = 0$ (i=n_1+1…N), Eq.(5) can be rewritten as follows.

$$\begin{Bmatrix}\mathbf{Q}_{J\lambda1}\\ \mathbf{Q}_{J\lambda2}\end{Bmatrix} = \begin{Bmatrix}\mathbf{Q}_{T\lambda1}\\ \mathbf{Q}_{T\lambda2}\end{Bmatrix} + \begin{bmatrix}\mathbf{F}^D_{\lambda1} & \mathbf{F}^D_{\lambda2}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{Bmatrix}\mathbf{Q}_{J\lambda1}\\ \mathbf{Q}_{J\lambda2}\end{Bmatrix} \tag{6}$$

Namely, Eq.(6) is rewritten as following equations.

$$\mathbf{Q}_{J\lambda1} = \mathbf{Q}_{T\lambda1} + \mathbf{F}^D_{\lambda1}\mathbf{Q}_{J\lambda1} + \mathbf{F}^D_{\lambda2}\mathbf{Q}_{J\lambda2} \tag{7.a}$$

$$\mathbf{Q}_{J\lambda2} = \mathbf{Q}_{T\lambda2} \tag{7.b}$$

where $\mathbf{Q}_{\lambda1}$ and $\mathbf{Q}_{\lambda2}$ are the block vectors, $\mathbf{Q}_{\lambda1} = \begin{Bmatrix}Q_{1,\lambda}\\ \vdots \\ Q_{n_1,\lambda}\end{Bmatrix}$ and $\mathbf{Q}_{\lambda2} = \begin{Bmatrix}Q_{n_1+1,\lambda}\\ \vdots \\ Q_{N,\lambda}\end{Bmatrix}$, $\mathbf{F}^D_{\lambda1}$ and $\mathbf{F}^D_{\lambda2}$ are the block matrix, $\mathbf{F}^D_{\lambda1} = \begin{bmatrix}F^D_{1,1,\lambda} & \cdots & F^D_{n_1,1,\lambda}\\ \vdots & \ddots & \vdots\\ F^D_{1,n_1,\lambda} & \cdots & F^D_{n_1,n_1,\lambda}\end{bmatrix}$ and $\mathbf{F}^D_{\lambda2} = \begin{bmatrix}F^D_{n_1+1,1,\lambda} & \cdots & F^D_{N,1,\lambda}\\ \vdots & \ddots & \vdots\\ F^D_{n_1+1,n_1,\lambda} & \cdots & F^D_{N,n_1,\lambda}\end{bmatrix}$. When the albedo or diffuse reflectivity of element, $\Omega^D_{i,\lambda}$, is 0, the diffuse radiosity, $Q_{J,i,\lambda}$, is obtained by Eq.(7.b). Then, transposing $\mathbf{F}^D_{\lambda1}\mathbf{Q}_{J\lambda1}$ from the right-hand side to the left-hand side in Eq.(7.a) and substituting Eq.(7.b) into Eq.(7.a) yields the following equation.

$$\left[\mathbf{I} - \mathbf{F}^D_{\lambda1}\right]\mathbf{Q}_{J\lambda1} = \mathbf{Q}_{T\lambda1} + \mathbf{F}^D_{\lambda2}\mathbf{Q}_{T\lambda2} \tag{8}$$

Eq.(8) is solved by the BiCGSTAB method, then diffuse radiosity, $Q_{J,i,\lambda}$, is obtained. Thus, the size of the linear equations to be solved is reduced to n_1 from N. After this procedure, $Q_{J,i,\lambda}$ is substituted into the following equations derived by Eq.(4), and spectral net rate of heat generation, $Q_{X,i,\lambda}$, can be calculated.

$$\mathbf{Q}_{X\lambda} = \mathbf{Q}_{T\lambda} - \mathbf{F}_{\lambda}^{A}\mathbf{Q}_{J\lambda} \tag{9}$$

The total heat flux of a surface element or the heat flux divergence of a volume element is obtained by

$$q_{X,i} = \frac{Q_{X,i}}{V_i} = \frac{1}{V_i}\int_0^{\infty} Q_{X,i,\lambda} d\lambda \tag{10}$$

in which, V_i=A_i for surface element.

The radiation elements used in the present study are composed of arbitrary triangles, quadrilaterals, tetrahedrons, wedges, and hexahedrons. The view factors are calculated by using ray tracing method based on the ray emission model as described by Maruyama and Aihara [5]. The number of rays from each element is set to 561 in this paper.

To determine the spectral absorption coefficients of CO_2 and H_2O, the Elsasser narrow band model is used in conjunction with the correlation parameters in Edwards wide band model as described by Guo and Maruyama [6]. The number of spectral partition is set to 100 for nongray calculations. Particles are assumed to be carbon spheres with a diameter of 30 μm in the present study. Employed phase function and the values of scattering and extinction efficiencies are listed by Tong and Skocypec [3]. To evaluate anisotropic scattering, the anisotropic scattering phase function reduced to zeroth-order delta function approximation [8] was used.

RESULTS AND DISCUSSIONS

Inhomogeneous gray medium in the cubic enclosure Radiative heat transfer in a unit cube with inhomogeneous gray medium is investigated and compared with the results of the other solutions [9]. Isotropic scattering is assumed in this subsection. The geometry is shown in Fig.1, in which L=H=W=1. All the walls are black and cold. Blackbody emissive power is given as unity in the medium. Distribution of the optical thickness; $\tau = \beta_\lambda$ (extinction coefficient) $\times$ W (the side length), is given as follows.

$$\tau = 0.9\left(1-\frac{|x|}{0.5}\right)\left(1-\frac{|y|}{0.5}\right)\left(1-\frac{|z|}{0.5}\right)+0.1 \tag{11}$$

The scattering albedo is set to 0.9.

The cubic medium was divided into n_pxn_pxn_p. The partition number n_p is varied from 3 to 19. In the case of n_p=9 and n_p=19, the distributions of surface heat flux along the line of x=-0.5, y=0 and divergence of heat flux along the line of y=z=0 are shown in Fig.2 (a) and (b), respectively. It is seen that the influence of volume element mesh on the heat flux is small. The solutions of Monte Carlo [9] and YIX method [9] are also shown in Fig.2 (a) and (b). The present solutions agree with Monte Carlo solutions and YIX solutions.

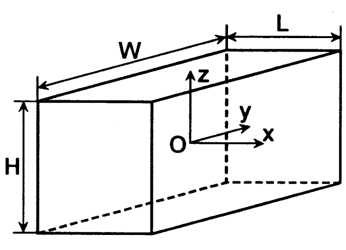
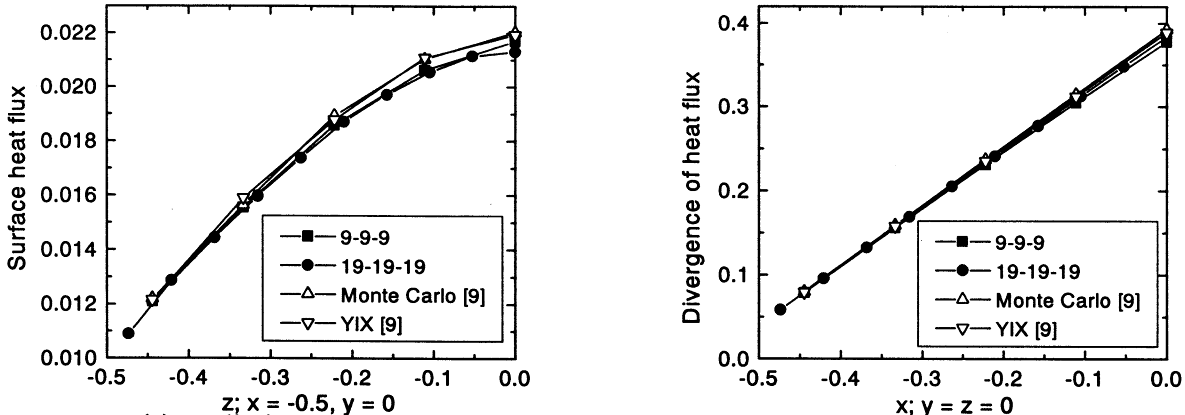
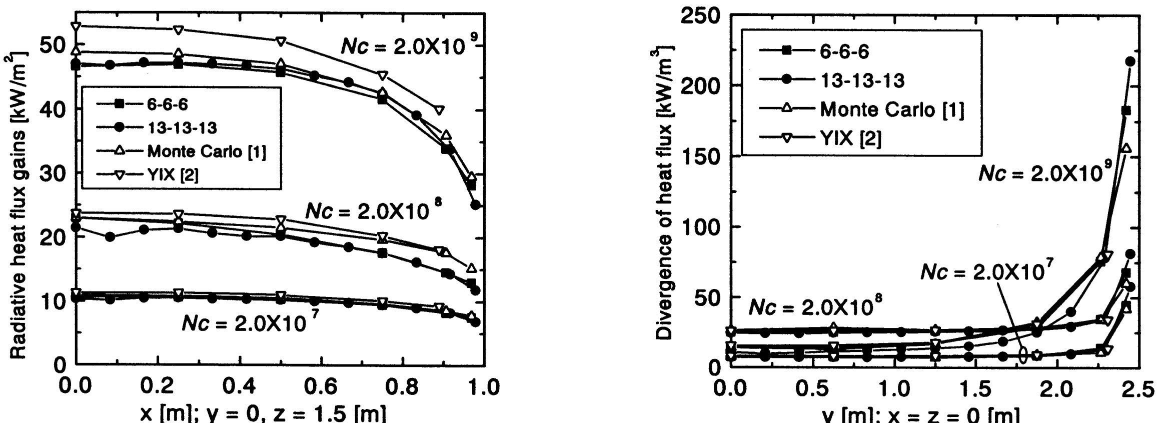


Figure 1. Geometry of a three-dimensional rectangular medium



(a) Radiative heat flux (b) Divergence of radiative heat flux

Figure 2. Comparison of solutions in inhomogeneous gray medium



(a) Radiative heat flux (b) Divergence of radiative heat flux

Figure 3. Comparison of solutions in nongray medium

Homogeneous nongray medium in the rectangular enclosure The accuracy of the present method is then verified for three-dimensional nongray and anisotropic scattering medium. The geometry is also shown in Fig.1, and L=2[m], H=3[m], and W=5[m]. The medium is a mixture of CO_2 and N_2 gases and carbon particles. The pressure of the total mixture is specified at 1atm, and a volume fraction of CO_2 is 0.21. The temperature of the medium is assumed at 1000K and the enclosure is black and cold surfaces. In this subsection, a one-eighths symmetric analysis model is used by considering three pure specular surfaces, Ω^S=1. The rectangular medium was divided into $n_p \times n_p \times n_p$. The analysis model includes $n_p \times n_p \times n_p$ volume elements, $3 \times n_p \times n_p$ black surface elements and $3 \times n_p \times n_p$ perfect mirror elements.

The distributions of surface heat flux along the line of y=0[m], z=1.5[m] and divergence of heat flux along the line of x=z=0[m] for three levels of carbon particle density are shown in Fig.3 (a) and (b), respectively. The solutions of Monte Carlo [1] and YIX method [2] are also shown in Fig.3.

The heat fluxes at the boundary agree with that by Monte Carlo and YIX method for all three levels of carbon density, approximately. However, the difference of heat flux between the present method and Monte Carlo method becomes larger at the vicinity of boundaries. In the case of n_p=13, it is shown that the heat flux by the present method is lower than the other solutions near the center of the medium ($0.0 \le x \le 0.2$[m]) with intermediate optical thickness (Nc=2.0×10^8[particles/m^3]). This may be an influence of symmetric surfaces (x=0[m]). For a carbon particle density of 2.0×10^9 particles/m^3, the heat flux results obtained by the YIX method are slightly higher than those by the other methods.

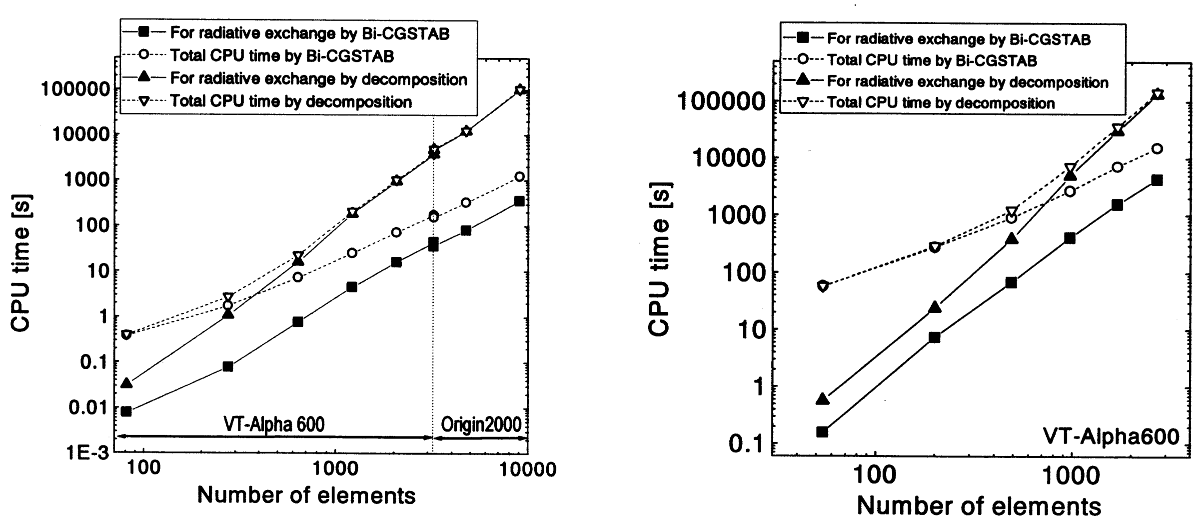
The differences of flux divergence by the present method basically agree with the two solutions except the vicinity of the boundaries in Fig.3 (b). The difference of the radiative transfer predictions between any two solutions of the present method, Monte Carlo method and YIX method is much larger in nongray medium than in gray medium. This may be attributed to the different spectral integration techniques used in the three different methods as noticed by Hsu and Farmer [9].

Comparison of a CPU time for cubic and rectangular mediums The relationship between CPU time and the number of surface elements for the cubic gray medium and rectangular nongray medium are shown in Fig.4 (a) and (b), respectively. The CPU time for calculating radiative exchange except the ray tracing and the total CPU time by the present method were compared with those by the previous radiation element method using the *LU* decomposition method. A personal computer VT-Alpha 600 was used as the calculating machine. However, when the volume divided number n_p is greater than or equal to 13, CPU time calculated by Origin2000 was shown as reference in Fig.4 (a).

In the case of the gray medium, CPU time for calculating radiative exchange by the previous method using the decomposition method is proportional to the 2.9th - 3.8th power of the number of the elements, as shown in Fig.4 (a). The CPU time increases if more elements are used. On the other hand, the CPU time for calculating radiative exchange by the present method is proportional to the 1.9th - 2.8th power of the number of the elements. In the case of 3211 elements, the CPU time for calculating radiative exchange by the present method was reduced to 1/84 and total CPU time was reduced to 1/22 compared with that by the previous method. Due to the remarkable reduction of the CPU time using the present method, determination of view factors consumes a large portion of calculation time. In the present case using the BiCGSTAB method, 25% of the CPU time was spent for solving radiative exchange, whereas most of the time was consumed for solving radiative exchange in the previous method using a direct method.

In the case of the nongray medium, Fig.4 (b) shows that CPU time for calculating radiative exchange by present method was reduced, too. Reduction of the total CPU time for nongray medium was smaller than that for gray medium because calculation of absorption, scattering, and extinction coefficient of gas and particles consumes a longer CPU time. However, the reduction in the CPU time becomes remarkable when the number of radiation elements increases.

Radiative heat transfer in a boiler model As a practical example, radiative heat transfer in a boiler model as shown in Fig.5 is investigated. Combustion gas is composed of CO_2, H_2O, and N_2, and the total pressure is 1atm. Mole fractions of CO_2 and H_2O are assumed to be 0.119 and 0.085, respectively. Temperature of the boiler wall is 623K and that of the gas exit is 813K. All walls are diffuse with emissivity of 0.8. Temperature profile inside the boiler is assumed as follows as in the study of Guo et al. [6].



(a) Gray medium

(b) Nongray medium

Figure 4. Comparison of CPU time

$$T = T_0 \left[a_1 \left(1 - \frac{|2x - W_1|}{2W - W_1} \right) \left(1 - \frac{|2y - L|}{2L} \right) \left(1 - \frac{|z - H_1|}{H - H_1} \right) + b_1 \right] \quad \text{(at } z \geq H_1\text{)}$$

$$T = T_0 \left[a_2 \left(1 - \frac{|2x - W|^3}{W^3} \right) \left(1 - \frac{|2y - L|}{10L} \right) \left(1 - \frac{|2z - H_1|}{10H_1} \right) + b_2 \right] \quad \text{(at } z < H_1\text{)} \qquad (12)$$

in which, a_1=1, b_1= 2.5, a_2=2.8, b_2=1, and the input fuel temperature T_0=563K. Carbon particles in the present model are assumed to be produced uniformly in a region where 1000K ≤ T ≤ 2000K and $H_1/9 \leq z \leq 6H_1/9$. Three levels of carbon density (Nc=2.0×10^7, 2.0×10^8, 2.0×10^9 [particles/m^3]) are studied. In actual calculation, a half analysis model is employed since the boiler is symmetric along the center plane of $y=L/2$. The analysis model of boiler is comprised of 1071 elements.

The distributions of normalized heat flux ($q_x/\sigma T_n^4$, T_n=2000K) at the wall are illustrated in Fig.6 for three levels of the carbon density. It is seen that larger heat fluxes are distributed at the wall near the flame region, and that the heat flux at the wall near the flame is strongly influenced by the particle density. The larger particle density is, the higher the heat flux becomes.

Regarding the boiler model, CPU time for calculating radiative exchange was also reduced by the present method. The CPU time for calculating radiative exchange in the case of Nc=2.0×10^9 was 216 seconds, and it was 30 times shorter than that by the previous method for three levels of the carbon density. The total CPU time by the present method was 2639 seconds, and it was 3.4 times shorter than that by the previous method. When we solve a problem with a large number of radiation elements, this reduction rate in total CPU time can be improved. Further more improvement can be expected by improving the ray tracing method in homogeneous nongray participating media.

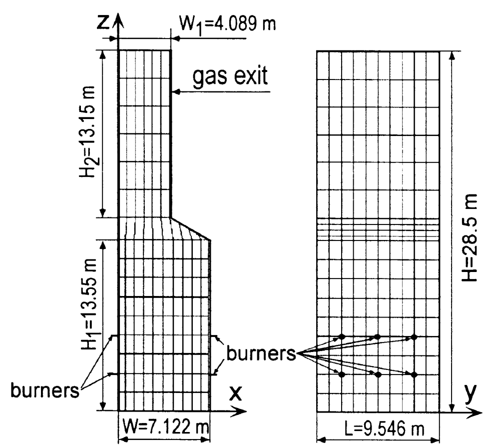


Figure 5. Geometry of a three-dimensional boiler furnace

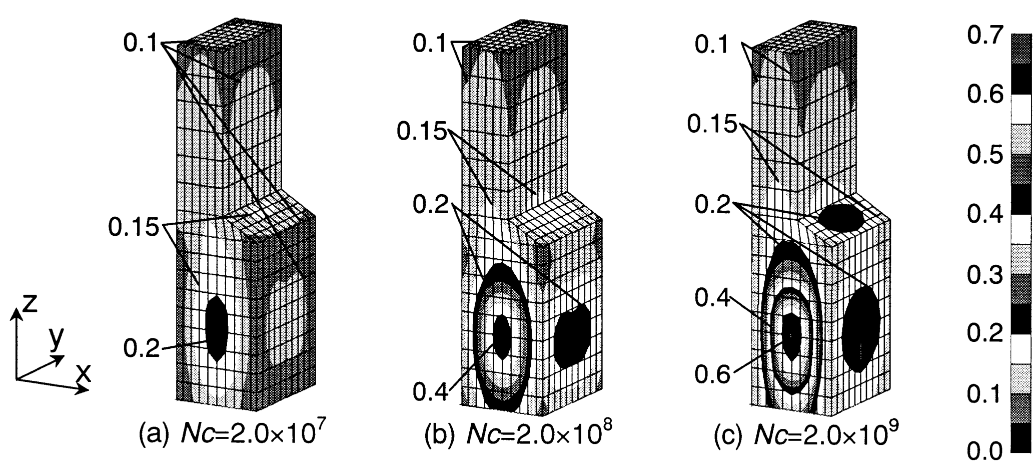


Figure 6. Distributions of normalized radiative heat flux at the wall

CONCLUSIONS

To reduce the time for calculating radiative heat transfer comprised of a large number of radiation elements, the radiation element method by ray emission model (REM^2) was improved by reduction of the size of equations to solve and using the stabilized bi-conjugate gradient (BiCGSTAB) method for solving equations.

Total CPU time was reduced to 1/22 for a model comprised of 3211 volume and surface elements, compared with the previous method using a decomposition method to solve linear equations. The calculation time using the present iterative method (BiCGSTAB) is proportional to the 1.9th - 2.8th power of the number of radiation elements, whereas it is proportional to the 2.9th - 3.8th power of the number of elements in the case of the previous method (*LU* decomposition). More reduction in the CPU time can be expected for a large-scale calculation. If an analysis model contains many elements without scattering coefficient or diffuse reflectivity, such as pure gas volume elements and black and pure specular surfaces, the present method can much reduce a calculation time, because the size of the matrix to solve is reduced.

REFERENCES

1. Farmer, J. T. and Howell, J. R., Monte Carlo Prediction of Radiative Heat Transfer in Inhomogeneous, Anisotropic, Nongray Media, *J. Thermophysics & Heat Transfer*, Vol.8, No.1, pp. 133-139, 1994.

2. Hsu, P. F., Tan, Z. and Howell, J. R., Radiative Transfer by the YIX Method in Nonhomogeneous, Scattering, and Nongray Media, *J. Thermophisics & Heat Transfer*, Vol.7, No.3, pp.487-495, 1993.

3. Tong, T. W. and Skocypec, R. D., Summary on Comparison of Radiative Heat Transfer Solutions for a Specified Problem, *Developments in Radiative Heat Transfer, ASME*, HTD-Vol.203, pp. 253-258, 1992.

4. Yang, W. J., Taniguchi, H. and Kudo, K., Radiative Heat Transfer by the Monte Carlo Method, *Advances in Heat Transfer*, Vol.27, chap.4, 1995.

5. Maruyama, S. and Aihara, T., Radiation Heat Transfer of Arbitrary Three-Dimensional Absorbing, Emitting and Scattering Media and Specular and Diffuse Surfaces, *Trans. ASME, J. Heat Transfer*, Vol.119, pp.129-136, 1997.

6. Guo, Z. and Maruyama, S., Radiative Heat Transfer in Inhomogeneous, Nongray, and Anisotropically Scattering Media, *Int. J. Heat Mass Transfer*, Vol.43, pp.2325-2336, 2000.

7. Oguni, T., Murata, K., Miyoshi, T., Dongarra, J. J. and Hasegawa, T., *Microcomputer - Aided determinant Calculation (with Floppy disk)*, chap.10, Maruzen, 1991 (in Japanese).

8. Maruyama, S., Radiative Heat Transfer in Anisotropic Scattering Media with Specular Boundary Subjected to Collimated Irradiation, *Int. J. Heat Mass Transfer*, Vol.41, pp.2847-2856, 1998.

9. Hsu, P. F. and Farmer, J. T., Benchmark Solution of Radiative Heat Transfer within Nonhomogeneous Participating Media Using the Monte Carlo and YIX Method, *Trans. ASME, J. Heat Transfer*, Vol.119, pp.185-188, 1997.