Determination of workpiece flow stress and friction at the 
chip–tool contact for high-speed cutting

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Abstract

This paper presents a methodology to determine simultaneously (a) the flow stress at high deformation rates and temperatures that are encountered in the cutting zone, and (b) the friction at the chip–tool interface. This information is necessary to simulate high-speed machining using FEM based programs. A flow stress model based on process dependent parameters such as strain, strain-rate and temperature was used together with a friction model based on shear flow stress of the workpiece at the chip–tool interface. High-speed cutting experiments and process simulations were utilized to determine the unknown parameters in flow stress and friction models. This technique was applied to obtain flow stress for P20 mold steel at hardness of 30 HRC and friction data when using uncoated carbide tooling at high-speed cutting conditions. The average strain, strain-rates and temperatures were computed both in primary (shear plane) and secondary (chip–tool contact) deformation zones. The friction conditions in sticking and sliding regions at the chip–tool interface are estimated using Zorev’s stress distribution model. The shear flow stress \(k_{chip}\) was also determined using computed average strain, strain-rate, and temperatures in secondary deformation zone, while the friction coefficient \(\mu\) was estimated by minimizing the difference between predicted and measured thrust forces. By matching the measured values of the cutting forces with the predicted results from FEM simulations, an expression for workpiece flow stress and the unknown friction parameters at the chip–tool contact were determined. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Orthogonal cutting; High-speed cutting; FEM simulation of cutting; Flow stress modeling; Friction at high-speed cutting

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1. Introduction

High-speed cutting (HSC) of hard alloy steels (up to hardness of 62 HRC) offers several advantages such as reduction of finishing operations, elimination of part distortion, achievement of high metal removal rates and lower machining costs as well as improved surface integrity [1]. However, HSC results in high temperatures and stresses at the tool–workpiece interface. Consequently, cost-effective application of this technology requires a fundamental understanding of the relationships between process variables and cutting conditions. Thus, it is necessary to understand how temperatures and stresses, developed during HSC, influence tool wear and premature tool failure as well as residual stresses on machined surfaces.

In machining hard materials, continuous (Type II) chip formation is observed at conventional to high cutting speeds and low to moderate feed rates. However, at higher feed rates “saw tooth” or “shear localized” (Type IV) chips are produced [2]. The latter type of chip formation can cause cyclic variations of both cutting and thrust forces and can result in high frequency vibrations that
affect tool life and failure [3]. Throughout the scope of this work, only the cutting conditions that result in continuous chip formation have been considered. Therefore, the mechanics and simulation of saw tooth (shear localized) chip formation is not discussed in this paper.

2. Background in orthogonal metal cutting

Metal cutting process can be considered as a deformation process where deformation is highly concentrated in a small zone [4]. Thus, cutting can be considered as a chip formation process and simulated using Finite Element Method (FEM) techniques. The main advantage of such an approach is to be able to predict chip flow, cutting forces, tool temperatures and stresses that result in various cutting conditions. However, material flow characteristics, or flow stress, at high temperature and deformation rates are required to conduct such predictions. There are very few material flow stress data available for the deformation conditions that exist in machining. Those data are mainly obtained by using impact compression tests for various materials at the moderate deformation rates [5]. However, further data is needed to overcome the uncertainty in the high temperature and strain rate material property data suitable for simulation of high-speed cutting.

Classic orthogonal cutting model for continuous chip formation assumes plane-strain deformation conditions. Basic representation of the process model is illustrated in Fig. 1. Geometric relations in orthogonal cutting model yield the following equations for normal and frictional forces, \( F_n \) and \( F_f \), on the tool rake face as functions of measured cutting force components, \( F_c \) and \( F_t \), and tool rake angle as:

![Fig. 1. Forces generated during orthogonal cutting process.](image-url)
\[ F_n = F_c \cos \alpha - F_t \sin \alpha \] (1)
\[ F_f = F_c \sin \alpha + F_t \cos \alpha \] (2)

In conventional machining at low cutting speeds, the friction mechanism is mostly effective at the tool flank face. However, in high-speed machining due to tremendous increase in the chip velocity the chip–tool contact friction is much more significant at the tool rake face [6]. As a result, the increasing sliding velocity and frictional stress cause significant wear on the tool rake face. Therefore, the rate of the tool wear in high-speed machining depends heavily on the chip–tool contact conditions.

2.1. Stress distributions on the tool rake face

The normal (\( \sigma_n \)) and frictional shear (\( \tau_f \)) stress distributions at the chip–tool interface characterize the cutting temperature and tool wear. Those distributions are commonly represented as shown in Fig. 2 [7]. According to Zorev [7], two regions exist simultaneously on the tool rake face when machining under dry conditions. From the tip of the tool up to a point, frictional stress is considered constant in a sticking region. After this point, frictional stress decreases on the tool rake face in a sliding region where Coulomb’s friction law can be applied.

Fig. 2. Curves representing normal (\( \sigma_n \)) and frictional (\( \tau_f \)) stress distributions on the tool rake face [7].
2.2. Friction characteristics on the chip–tool interface

The contact regions and the friction parameters between the chip and the tool are influenced by factors such as cutting speed, feed rate, rake angle, etc. [8], mainly because of the very high normal pressure at the surface. The prevalent conditions at the chip–tool interface constrain the use of the empirical values of the coefficient of friction found from ordinary sliding test conditions. While numerous works have been reported to quantitatively explain the variable and high values of friction at the rake face, none has provided reliable quantitative predictive models that are devoid of experimental testing.

2.3. Flow stress models for workpiece materials

The flow stress or instantaneous yield stress at which workpiece material starts to flow is mostly influenced by temperature \( T \), effective strain \( \bar{\varepsilon} \), effective strain-rate \( \dot{\varepsilon} \), and microstructure \( S \), i.e. chemical composition, phases, grain size. Thus, the flow stress \( \bar{\sigma} \) can be given as:

\[
\bar{\sigma} = f(T, \bar{\varepsilon}, \dot{\varepsilon}, S)
\]

(3)

For practical cutting speeds in machining, the average strain-rate in the shear zone lies in the range of \( 10^3 \) to \( 10^5 \) sec\(^{-1} \) or even higher [4]. These values are much higher than the strain rates of \( 10^{-2} \) to \( 10^{-1} \) sec\(^{-1} \) that are normally encountered in compression and tension tests. Many researchers developed several techniques to determine the flow stress of metals at high strain-rates and temperatures [9–11]. Some assumed that for a particular strain-rate and temperature combination the relationship between the effective flow stress and effective strain for the work material considered varying with strain-rate \( \dot{\varepsilon} \) and temperature [12]. Later Oxley used velocity-modified temperature that consist of strain-rate and temperature for carbon steels [4].

Some used an impact compression material testing machine to measure flow stress at temperatures 20–1000°C and strain-rates 200–2000 sec\(^{-1} \) [12]. Empirical expressions for the flow stress including the history effects of the temperature and strain-rate without anneal softening and age hardening effects are also developed [5]. As an example, an expression for the flow stress of cold work mold steel (Cr–Mo steel) is here given [13]:

\[
\bar{\sigma} = A(10^{-3}\dot{\varepsilon})^M e^{kT} (10^{-3}\dot{\varepsilon})^k T \left[ \int_{T, \bar{\varepsilon} = \dot{\varepsilon}} e^{-kT/N(10^{-3}\dot{\varepsilon})-mN \bar{\varepsilon}} \, d\bar{\varepsilon} \right]^N
\]

(4)

where,

\[
A(T) = 1.46\exp(-0.0013T) + 0.196\exp(-0.000015(T-400)^2) - 0.0392\exp(-0.01(T-100)^2)
\]

\[
N(T) = 0.162\exp(-0.0017) + 0.092\exp(-0.0003(T-380)^2)
\]

\[
M(T) = 0.047, k = 0.000065, m = 0.0039
\]
3. Modeling of high-speed orthogonal cutting process

Earlier models of metal cutting were based on only basic shear plane assumption or slip line field analysis [14,15]. Friction conditions at the chip–tool interface in early FEM models of metal cutting were ignored [16] or assumed to be constant with a coefficient of friction based on Coulomb’s law at the chip–tool interface [17]. Usui [12] modeled the tool–chip interaction with a frictional force as a function of normal force as boundary conditions using the shear flow stress of the workpiece at the primary zone obtained from experiments. Iwata [18] incorporated the frictional stress as a function of normal stress at the boundary conditions with an empirical relationship based on a coefficient of friction obtained from friction tests. Shih [19] used a constant coefficient of friction in the sticking region at the chip–tool interface and a coefficient linearly decaying to zero in the sliding region. Later, models that include both chip–tool contact friction and material behavior at high strains, strain-rates and temperatures were proposed [4] and noteworthy attempts for FEM simulation of cutting processes were presented [20–23]. The FEM approach proposed by Marusich et al. has led to a commercially available metal cutting simulation software which appears to be successful [22].

Recently, orthogonal cutting was also simulated using a software developed for large plastic deformations, DEFORM™ and chip formation for continuous and segmented chips were predicted using a fracture criteria [24]. Capabilities in generating a very dense mesh near the tool tip and remeshing adaptively makes this software applicable to simulate the cutting process. Although the assumed input data for material properties and friction were quite approximate, simulation of metal cutting was carried out with relatively little effort [25]. These preliminary investigations demonstrated that with reliable input data on material properties it is possible to estimate chip flow and cutting forces. In addition, this model was also extended to simulate chip flow in 2-D flat end milling with straight cutting edges and verified with experiments [26]. Work is in progress to extend this approach to estimate temperatures and stresses in cutting operations involving 3-D chip flow, e.g. turning and end milling with nose radius tools [27].

4. Methodology to determine flow stress and friction at chip–tool interface

In HSC, extremely high strain-rates (about 1.67×10^5 sec^{-1} at 500 m/min cutting speed and 0.05 mm undeformed chip thickness) and temperatures (about 1400°C) at the chip–tool interface occur in the primary deformation zone and secondary deformation zone, respectively. To address the issues of flow stress and friction, a methodology was developed for determining simultaneously both the flow stress of workpiece material and the friction conditions at the chip–tool contact interface.

The basic concept of the proposed methodology is the use of orthogonal cutting experiments and FEM simulations in order to determine the flow stress and friction conditions used for the range of high-speed cutting. Therefore, a limited number of orthogonal end turning experiments on P20 mold steel disks (at hardness of 30 HRC) was conducted using uncoated tungsten carbide (WC) tooling (Fig. 3). From the experiments, two components of cutting force (F_c and F_t), chip thickness (t_c), and chip–tool contact length (l_c) were measured. In addition, the microscopic pic-
tures of chips were collected to identify chip formation. Later, FEM simulations of continuous chip flow in orthogonal cutting process were conducted.

4.1. High-speed orthogonal cutting experiments

The purpose of these experiments was to obtain cutting force data for a process model of orthogonal cutting.

In these orthogonal cutting experiments Fig. 3, the following conditions were used:

- Workpiece: P20 mold steel disks, 3 mm thickness, 30 HRC hardness
- Tool: uncoated carbide (WC) inserts, rake angle $\alpha=-7^\circ$, edge radius $\rho=0.012$ mm
- Cutting speed, $V_c$: 200, 300 and 550 m/min
- Feed rate, $V_f$: 0.025, 0.051, 0.075 and 0.100 m/rev

Experiments were replicated twice at each cutting condition in order to minimize experimental errors. A sudden tool failure occurred during experiments at a cutting speed of 550 m/min and feed rates of 0.075 and 0.100 mm/rev.
4.2. Chip formation

The shape and geometry of the chips were investigated using the optical-microscope Fig. 4. In this analysis, the chip shapes were mostly continuous (Type II). However, saw-tooth shaped chips were observed at conditions with cutting speed of 300 m/min and undeformed chip thickness of 0.100 mm. Nevertheless, it can be stated that the chip formation indicates continuous type of chips at the lower feed rates (<0.100 mm/rev) and at the lower cutting speeds (<550 m/min) when high-speed cutting of P20 mold steel. This observation supports the shear localization behavior of hard steels in the secondary deformation zone due to runaway thermo-mechanical deformation at

Fig. 4. Chip geometry measured from the experiments in orthogonal cutting of P20 mold steel.
high feed and cutting speeds postulated by Zhen-Bin et al. [28]. Furthermore, it is in direct opposition to the brittle fracture hypothesis postulated by Elbestawi et al. [29]. It should be noted that simulation of shear localized chip formation is not studied in this work.

4.3. Cutting force measurements

The cutting force \( F_c \) in the cutting direction (along \( z \)-axis) and thrust force \( F_t \) in the feed direction (along \( y \)-axis), Fig. 3, were measured during orthogonal turning of P20 mold steel disks. In each experiment, a fresh part of the cutting tool was used and experiments were replicated twice at each cutting condition in order to reduce experimental error. In Fig. 5, measured cutting forces \( F_c \) and thrust forces \( F_t \) per unit width of cut are presented. Although a significant increment in cutting forces with the increase of undeformed chip thickness was found, this behavior was rather insignificant for thrust forces. This is due to high and consistent friction contact between chip and tool at the tool rake face. However, it should be noted that additional replication of the experiments may change the average values of cutting forces as opposed to the average of two replications.

4.4. Flow diagram of the methodology to determine flow stress and friction

For the process simulation of high-speed cutting of P20 mold steel (30 HRC) with uncoated carbide (WC) tooling, the flow stress data and friction model, described earlier, were used. At each of the cutting conditions, the chip flow was simulated, and cutting force \( F_c \), thrust force \( F_t \), deformed chip thickness \( t_c \) and chip–tool contact length \( l_c \) were predicted. The predictions were compared with measurements and the flow stress data and friction values were modified until an acceptable correlation could be obtained between experimental and predicted values. A detailed flow chart explaining this procedure is given in Fig. 6.

4.5. Material properties for process simulation of orthogonal cutting

Since chemical composition of P20 mold steel is similar to the Cr–Mo steel, the flow stress equation obtained for Cr–Mo steel was assumed to be also valid for P20 mold steel. Therefore, the data for Cr–Mo steel, given in Eq. (4), was used as initial flow stress [13]. Other data for input preparation of the process simulations is given in Table 1.

4.6. Estimation of friction conditions at chip–tool interface

Friction conditions at the chip–tool contact can be interpreted in terms of two frictional modes, which are represented by shear friction and friction coefficient relations Fig. 2. Constant shear stress law \( \tau = \sigma / \sqrt{3} = k_{\text{chip}} \) with Von Mises plastic flow criterion and friction coefficient, \( \mu = \tau / \sigma_n \) need to be properly defined on the sticking region \((0 \leq x \leq l_p)\) and the sliding region \((l_p \leq x \leq l_c)\), respectively.

An appropriate initial value for the friction coefficient \( \mu_{\text{p,initial}} \) was selected as the ratio of the frictional force \( F_f \) and normal force \( F_n \) acting on the tool rake face. Both forces were calculated from the measured force components at the given rake angle.
Fig. 5. Measured cutting ($F_c$) and thrust ($F_t$) forces per unit width and the influence of cutting speed.
Experimental data for orthogonal cutting: cutting force ($F_c$), thrust force ($F_t$) at different speeds and undeformed chip thickness

Choose a set of cutting conditions used in experiments

**Flow stress model:**
- Initial flow stress data at low strain and strain rates
- Assume flow stress is only temperature and strain rate dependent at high strain rates
- Flow stress is extrapolated for high strain rates initially

**Friction model:**
- Define variable friction coefficient for sticking and constant friction coefficient in sliding region
- Estimate initial value for $k_{chip}$
- Set $\mu = F_t / F_c$ for the sliding region
- Set $\mu = k_{chip} / \sigma_0$ for the sticking region until $\mu = \mu(\sigma_c)$

Process simulation in DEFORM-2D using same cutting conditions and with the above assumptions

Determine the average values of strain rate and temperature in the deformation zones from the simulation results

Modify flow stress for the determined strain rate and temperature regimes

Does the predicted cutting force ($F_c^*$) agree with measured cutting force?

NO

Do the predicted forces ($F_c^*$ and $F_t^*$) agree with the measured forces?

NO

YES

- Keep the modified flow stress data
- Reset the friction data used

1) Output a list of $\mu$ and $k_{chip}$ used in each simulated case
2) Output flow stress data, average strain rates, average temperatures for all cases
3) Identify coefficients of flow stress equation with least square method

ALL CASES DONE

YES

NO
Table 1
List of cutting conditions, material properties for P20 mold steel workpiece and uncoated carbide tool used in the process simulations

Orthogonal cutting conditions
Cutting speed, $V_c$, (m/min) 200, 300, 550
Feed, uncut/undeformed chip thickness, $t_u$, (mm) 0.025, 0.051, 0.075, 0.100
Width of cut (mm) 1

Properties of workpiece (P20 mold steel)
Emissivity ($\sigma$) 0.60 (100°C) 0.65 (500°C) 0.75 (1000°C)
Coefficient of thermal expansion ($10^{-6}/$°C) 1.3 (425°C) 1.4 (650°C)
Specific heat (J/kg/°C) 470
Thermal conductivity (W/m°C) 51.5
Poisson’s ratio ($\nu$) 0.3
Young’s modulus (GPa) 260

Geometry and properties of cutting tool (Tungsten Carbide F Grade)
Tool tip radius, $r$, (mm) 0.012
Rake angle, $\alpha$, (deg) 27
Clearance angle, $\gamma$, (deg) 15.3
Emissivity ($\sigma$) 0.5
Coefficient of thermal expansion ($10^{-6}/$°C) 5.2
Specific heat (J/kg/°C) 343.3
Thermal conductivity (W/m°C) 120
Poisson’s ratio ($\nu$) 0.22
Young’s modulus (GPa) 522 (20°C) 620 (100°C)

$$F_n,\text{ estimated} = (F_c,\text{ measured}) \cos \alpha - (F_t,\text{ measured}) \sin \alpha$$ (5)

$$F_t,\text{ estimated} = (F_c,\text{ measured}) \sin \alpha + (F_t,\text{ measured}) \cos \alpha$$ (6)

Therefore, the initial friction coefficient was calculated using measured cutting forces as follows:

$$\mu_{p,\text{ initial}} = \frac{F_t,\text{ estimated}}{F_n,\text{ estimated}}$$ (7)

This average friction coefficient was found between 0.5 to 0.7 except for the undeformed chip thickness of 0.025 mm which was about 1.0. The increase in the mean friction coefficient was due to so-called size effect of the cutting edge. At the undeformed chip thickness of 0.025 mm, the ratio of the cutting edge radius ($\rho=0.012$ mm) to the undeformed chip thickness was 0.5. It is well known that the higher this ratio is, the higher the specific cutting pressure, i.e. friction force.

When using the simulation software, DEFORM-2D™, the most appropriate way of implementing chip–tool contact friction is to use a variable friction coefficient that is a function of the normal pressure at the tool rake surface ($\mu_i=f(\sigma_n)$). Since the uniformly distributed shear frictional stress in the sticking region is known to be equal to the local shear flow stress ($\tau_i=k_{\text{chip}}$), the friction coefficient in the sticking region was defined as:

$$\mu_i = \mu_0 = \frac{k_{\text{chip}}}{\sigma_{\text{max}}}$$ at $x=0$ (8)
\[ \mu_i = \frac{k_{\text{chip}}}{\sigma_n} \quad 0 < x \leq l_p \]  

(9)

In the sliding region, a constant friction coefficient \( \mu_p \) was defined as:

\[ \mu_i = \mu_p \quad l_p < x \leq l_c \]  

(10)

Thus, this friction model includes two major unknowns, the local shear flow stress of the chip \( k_{\text{chip}} \) in the sticking region, and a constant friction coefficient \( \mu_p \) in the sliding region, used in the process simulations.

4.7. Computation of average strain rate and temperatures in deformation zones

In order to determine flow stress data by using proposed methodology, average strain-rates, strains and temperatures for primary and secondary deformation zones are computed after each process simulation for different cutting conditions. Figs. 7 and 8 indicate the primary and secondary deformation zones. Process simulation predicts the state variables such as strain, strain-rate, temperature and Von Mises stresses. By using the post processor files for element connectivity, coordinates of the nodes, defined boundary conditions, strain-rates, and node temperatures, the

Fig. 7. Predicted strain-rate distribution and identified primary and secondary regions in orthogonal cutting of P20 mold steel \( (V_c=550 \text{ m/min}, t_c=0.051 \text{ mm}) \).
deformation zones were identified using a computer program developed. In the identified primary and secondary deformation zones, average strain-rates \( \dot{e}_{\text{ave}} \) and temperatures \( T_{\text{ave}} \) were calculated as listed in Table 2. These values were calculated as area-weighted averages by extracting the simulated data and from the following equation:

\[
\dot{\phi}_{\text{ave}} = \frac{\sum_{i=1}^{n} \phi_i A_i}{\sum_{i=1}^{n} A_i}
\]

\( (11) \)

Table 2
Calculated average temperatures and strain-rates in the primary and the secondary deformation zones

<table>
<thead>
<tr>
<th>( V_c (\text{m/min}) )</th>
<th>( T_{\text{ave}} (^\circ \text{C}) )</th>
<th>( \dot{e}_{\text{ave}} ) (sec(^{-1}))</th>
<th>( T_{\text{ave}} (^\circ \text{C}) )</th>
<th>( \dot{e}_{\text{ave}} ) (sec(^{-1}))</th>
<th>( T_{\text{ave}} (^\circ \text{C}) )</th>
<th>( \dot{e}_{\text{ave}} ) (sec(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>535</td>
<td>60,746</td>
<td>736</td>
<td>36,031</td>
<td>444</td>
<td>52,857</td>
</tr>
<tr>
<td>300</td>
<td>544</td>
<td>120,448</td>
<td>762</td>
<td>40,596</td>
<td>550</td>
<td>88,175</td>
</tr>
<tr>
<td>550</td>
<td>560</td>
<td>157,625</td>
<td>788</td>
<td>71,190</td>
<td>581</td>
<td>122,098</td>
</tr>
</tbody>
</table>
where $\phi_i =$ predicted value of strain rate ($\dot{\varepsilon}_i$) and temperature ($T_i$), $A_i =$ area of the element $i$, and $n =$ number of elements in primary or secondary deformation zones.

The extent of primary deformation zone for calculation was defined as the region in which elemental strain-rate is higher than the value of nominal average strain rate ($\dot{\varepsilon}_n$) determined from cutting conditions as:

$$\dot{\varepsilon}_n = \frac{V_c}{t_u}$$

(12)

At the same cutting conditions, process simulations were conducted in an iterative scheme until predicted thrust and cutting force match with the experiments. The unknowns of the friction model, shear flow stress in the chip and the friction parameters, were found from iterations as listed in Table 3.

By using the determined flow stress and friction data, process simulations were conducted repeatedly and the predicted cutting forces were compared in Figs. 9 and 10. The prediction error in each simulation was set to be less than 10% for cutting and thrust force ($|\Delta F_c, F_t| \leq 0.10$).

<table>
<thead>
<tr>
<th>$V_c$(m/min)</th>
<th>$V_f=0.051$ mm/rev</th>
<th>$V_f=0.075$ mm/rev</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k_{chip}$ (MPa)</td>
<td>$k_{chip}$ (MPa)</td>
</tr>
<tr>
<td>200</td>
<td>907</td>
<td>915</td>
</tr>
<tr>
<td>300</td>
<td>911</td>
<td>932</td>
</tr>
<tr>
<td>550</td>
<td>923</td>
<td>954</td>
</tr>
</tbody>
</table>

where $k_{chip}$ is the shear flow stress in the chip and $\mu_0$ are the estimated friction coefficients for friction at chip–tool contact interface.
4.8. Identification of coefficients in flow stress expression

An empirical expression of flow stress, developed by Maekawa [5], was used in order to represent the flow stress data determined for high-speed machining conditions (cutting speed up to 550 m/min).

\[ \sigma = K(e^{a_T} + A e^{b(T-T_0)})^c \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_R} \right)^d \]  \hspace{1cm} (13)

In Eq. (13), the specified parameter \((\dot{\varepsilon}_R)\) is introduced to neutralize the strain-rate unit. The unknown coefficients, \(a, b, c, d, A, K,\) and \(T_0\), were identified by using determined flow stress data at temperatures of 20, 100, 500, 1000°C, strains 0.01, 0.1, 1, 10 mm/mm and strain-rates 0.1, 100, 1000, 50000 sec\(^{-1}\) by applying the least squares method to minimize the sum of the square of the error (see Appendix A). Therefore, the coefficients of flow stress equation for P20 mold steel were found as: \(K=1339.4, \ T_0=400, \ A=0.18268, \ \alpha=-0.0013, \ b=-0.00001, \ c=0.02964, \) and \(d=0.0363.\) The determined flow stress graph for P20 mold steel is presented in Fig. 11.

5. Conclusions

In this paper, a methodology is presented to determine simultaneously the flow stress of the workpiece material, and the friction at the chip–tool contact interface at high deformation rates,
temperatures encountered in the cutting zone. This methodology uses the cutting and thrust force data measured from high-speed orthogonal cutting experiments as reference in order to calibrate a simulated process model. The methodology was applied to obtain flow stress data for P20 mold steel (at hardness of 30 HRC) and friction data in machining with uncoated tungsten carbide (WC) cutting tool. The experimental cutting conditions of 200–550 m/min cutting speed ($V_c$), and 0.025–0.100 mm/rev feed rate ($V_f$) were used. From the process simulations, boundaries of the primary deformation zone were identified and the average strain-rates ($\bar{\varepsilon}_{ave}$) and temperatures ($T_{ave}$) in this region were computed. Flow stress of the workpiece material at computed average strain-rates and temperatures was determined using an iterative scheme until the difference between predicted and measured cutting forces becomes less than 10%. In addition, the average strain-rate and temperature were computed at the chip–tool contact site and the average shear friction ($k_{chip}$) was estimated. The unknown friction coefficients were determined in another iterative scheme until the difference between predicted and measured thrust forces was less than 10%. Finally, cutting force predictions from process simulations, using the determined flow stress and friction models, were compared with experiments and good agreements were obtained (Figs. 10 and 11).

Data for flow stress of the workpiece material and friction at the chip–tool interface, necessary as input into the FEM codes, are usually not available for the deformation rate and temperature regimes that exist in high-speed cutting conditions. Thus, it is possible to apply the methodology,
developed here, to other cutting conditions and workpiece/tool materials than those investigated in this paper. At present, this FEM based simulation technique is being extended to predict the tool temperature and stress distributions in turning and end milling processes with nose radius cutting tools.

Appendix A. Least squares estimation of coefficients for flow stress equation

Least squares method was used in estimating the unknown coefficients in the flow stress equation given as:

$$\sigma = K(e^{aT} + Ae^{b(T-T_0)^2}) \left( \frac{\dot{E}}{\dot{E}_{R}} \right)^c (\bar{E})^d \quad (A1)$$

While this flow stress model is not linear when it is used, by taking a log transform of Eq. (A1), the model becomes:

$$\log(\sigma) = \log K + \log(e^{aT} + Ae^{b(T-T_0)^2}) + c \log\left( \frac{\dot{E}}{\dot{E}_{R}} \right) + d \log(\bar{E}) \quad (A2)$$

where, $T_0=400^\circ C$ is used from the initial flow stress model, and $\dot{E}_{R}=10000 \text{ sec}^{-1}$ is used for cleaning units. The linear model with the error associated with each determined flow stress data $\delta_i$ included results in:

$$Y_i = Y_0 + xZ_i + \delta_i \quad (A3)$$

The linear least squares method is aimed at finding estimates of $\hat{Y}_0$ and $\hat{x}$ that minimize:

$$\sum_{i=1}^{n} \delta_i^2 = \sum_{i=1}^{n} (Y_i - \hat{Y}_0 - \hat{x}Z_i)^2 \quad (A4)$$

The solution can be written using a matrix notation.

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix}, \quad X = \begin{bmatrix} 1 & X_1 \\ 1 & X_2 \\ \vdots & \vdots \\ 1 & X_n \end{bmatrix}, \quad \Delta = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_n \end{bmatrix} \quad (A5)$$

The solution is found by differentiating the squared error in Eq. (A4) with respect to the model parameters. Setting the resulting linear equations equal to zero leads to final solution:

$$\hat{\beta} = \begin{bmatrix} \hat{Y}_0 \\ \hat{x} \end{bmatrix} = (X^TX)^{-1}(X^TY) \quad (A6)$$
References


